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AMERICAN MATHEMATICAL SOCIETY

EDITED BY

EARLE R. HEDRICK

DAVID R. CURTISS WILLIAM R. LONGLEY

WITH THE ASSISTANCE OF

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CAROLINE E. SEELY

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THE THANKSGIVING MEETING IN COLUMBUS

The two hundred ninety-fourth regular meeting of the Society was held at the Ohio State University on Friday and Saturday, November 27–28, 1931. About eighty persons attended the meeting, among whom were the following fifty-one members of the Society:

R. B. Allen, W. L. Ayres, F. R. Bamforth, G. M. Bareis, I. A. Barnett, W. D. Baten, H. L. Black, Henry Blumberg, R. S. Burington, L. E. Bush, A. D. Campbell, R. V. Churchill, Ben Dushnik, H. M. Gehman, W. C. Graustein, E. H. Hildebrandt, T. H. Hildebrandt, M. H. Ingraham, H. W. Kuhn, Lincoln LaPaz, C. I. Lubin, C. C. MacDuffee, E. J. McShane, G. M. Merriman, C. N. Moore, C. C. Morris, A. L. Nelson, R. L. Newlin, F. W. Owens, H. B. Owens, G. A. Parkinson, T. S. Peterson, H. S. Pollard, Saul Pollock, Tibor Radó, G. Y. Rainich, S. E. Rasor, C. E. Rhodes, S. A. Rowland, H. C. Shaub, Grace Shover, G. W. Starcher, P. M. Swingle, H. P. Thielman, T. O. Walton, J. H. Weaver, R. L. Wilder, F. B. Wiley, C. O. Williamson, B. F. Yanney, C. H. Yeaton.

On Friday evening a dinner was held at the Faculty Club. The dinner was attended by sixty persons.

On Friday afternoon Professor H. M. Gehman gave an address entitled *Homeomorphic geometry*. On Saturday morning Professor W. C. Graustein spoke to the Society on *Parallelism of Levi-Civita in classical differential geometry*. These addresses were delivered upon invitation of the Program Committee.

The meetings were held in the Administration Building and the Faculty Club on the third floor was made available for social purposes. On Saturday afternoon, upon rising vote, the thanks of those present were given to their hosts for the splendid arrangements made for this meeting.

Vice-President W. C. Graustein presided at the meeting on Friday afternoon, Professor H. W. Kuhn on Saturday morning, and Professor T. H. Hildebrandt on Saturday afternoon. The papers whose abstract numbers are followed by *t* were read by title. The paper of Professor Davis was read by Mr. Keller,

introduced by Professor Davis, and the paper of Professor Roth by Professor M. H. Ingraham. Professor Lanczos was introduced by Professor William Marshall, and Mrs. Miller by Professor P. R. Rider. Papers numbered 1–5 were read Friday afternoon, those numbered 6–11 Saturday morning, and papers numbered 12–26 Saturday afternoon.

1. *Variation problems of which the extremals are minimal surfaces*, by Professor Lincoln LaPaz. (Abstract No. 37–11–378.)

2. *On the Stieltjes integral*, by Dr. Ben Dushnik. (Abstract No. 37–11–379.)

3. *On the absolute convergence of the developments in Bessel's functions*, by Professor C. N. Moore. (Abstract No. 37–11–380.)

4. *Frequency laws for the sum of n variables which are subject each to given frequency laws*, by Professor W. D. Baten. (Abstract No. 37–11–381.)

5. *Polynomial approximation by the method of least squares*, by Professor H. T. Davis. (Abstract No. 37–11–382.)

6. *On the determination of formulae for the orders of line loci in five-space by graphic representation*, by Professor Saul Pollock. (Abstract No. 37–11–383.)

7. *On convex regions*, by Professor Tibor Radó. (Abstract No. 37–11–384.)

8. *On the linking of Jordan continua in E_n by $(n-2)$-cycles*, by Professor R. L. Wilder. (Abstract No. 37–11–385.)

9. *On a certain neighborhood property*, by Professor W. L. Ayres. (Abstract No. 38–1–1.)

10. *Covariant form of a special case of Dirac's equations*, by Professor G. Y. Rainich. (Abstract No. 37–11–386.)

11. *The electromagnetic field as a natural property of Riemannian geometry*, by Professor Cornelius Lanczos. (Abstract No. 37–11–387.)

12. *Class number in a linear associative algebra*, by Miss Grace Shover. (Abstract No. 37–11–388.)

13. *Note on the discriminant matrix of an algebra*, by Dr. L. E. Bush. (Abstract No. 37–11–389.)

14. *On the equivalence of the decompositions of an algebra with respect to a principal idempotent*, by Dr. L. E. Bush. (Abstract No. 37–11–390.)

15. *On algebraic equations having only real roots*, by Professor W. E. Roth. (Abstract No. 37–11–391.)

16. *Canonical forms for symmetric linear vector functions in pseudo-euclidean space*, by Professor R. V. Churchill. (Abstract No. 37–11–392.)

17. *Four points of intersection of lines drawn from the vertices of a triangle to the points of contact of the nine point circle with the in- and ex-circles*, by Mrs. Pearl C. Miller. (Abstract No. 37–11–393.)

18. *The symmetric* (n, n)*-correspondence and some geometric applications*, by Professor Arnold Emch. (Abstract No. 37–11–394–*t*.)

19. *An integral addition theorem for Bessel functions*, by Mr. H. P. Thielman. (Abstract No. 38–1–2.)

20. *Riquier's existence theorems*, by Professor J. M. Thomas. (Abstract No. 37–11–395–*t*.)

21. *Fourier developments for certain pseudo-periodic functions in two variables*, by Professor M. A. Basoco. (Abstract No. 37–11–396–*t*.)

22. *On finite and infinite completely monotonic sequences*, by Dr. I. J. Schoenberg. (Abstract No. 37–11–397–*t*.)

23. *Properties of coverings of abstract type related to reducibility*, by Professor E. W. Chittenden and Mr. Selby Robinson. (Abstract No. 37–11–398–*t*.)

24. *The linear graphs associated with electrical circuits*, by Mr. R. M. Foster. (Abstract No. 37–11–399–*t*.)

25. *On extending a continuous* (1–1) *correspondence of continuous curves on a sphere*, by Dr. V. W. Adkisson. (Abstract No. 37–11–400–*t*.)

26. *On the minors of absolutely convergent determinants*, by Professor L. W. Cohen. (Abstract No. 37–11–401–*t*.)

M. H. INGRAHAM,
Associate Secretary

THE NOVEMBER MEETING IN PASADENA

The two hundred ninety-fifth regular meeting of the Society was held at the California Institute of Technology in Pasadena on Saturday, November 28, 1931. The meeting was called to order at 10 a. m., and was presided over by Vice-President W. A. Manning. During an informal business session, Professor E. T. Bell presided. About fifty persons attended the meeting, including the following twenty-six members of the Society:

O. W. Albert, Harry Bateman, Clifford Bell, E. T. Bell, Myrtie Collier, P. H. Daus, Raymond Garver, H. E. Glazier, E. R. Hedrick, G. H. Hunt, D. H. Lehmer, W. A. Manning, A. D. Michal, R. L. Moore, A. E. Ross, A. A. Shaw, G. E. F. Sherwood, J. H. Sturdivant, S. E. Urner, H. C. Van Buskirk, L. E. Wear, M. G. Whiting, W. M. Whyburn, C. L. E. Wolfe, C. R. Worth, E. R. Worthington.

At the morning session, by invitation of the Program Committee, Professor R. L. Moore delivered an address entitled *A set of axioms for plane analysis situs*. This address will be published in full in an early issue of this Bulletin.

During the interval between the sessions, luncheon was served at the Atheneum of the California Institute. The program was completed at the afternoon session.

On motion of Professor W. A. Manning, it was voted, in view of the fact that the Summer Meeting of the Society is to be held on the Pacific Coast, that the meeting recommend to the Council of the Society that the usual Spring Meeting of the Society which was to have been held in 1932 at Stanford University be omitted. The acting secretary called attention to the fact that similar action has been taken to omit the usual 1932 June Meeting in the Pacific Northwest, and the usual November meeting in Los Angeles in 1932, in order to concentrate all the energies of this region on the coming Summer Meeting of the Society. It was pointed out also that abstracts of papers to be read at the Summer Meeting may be sent in at any time to Associate Secretary Ingraham, and may be printed in advance in the Bulletin in order to ensure priority.

In the absence of Associate Secretary T. M. Putnam, Professor E. R. Hedrick acted as Secretary.

The titles of papers read at the meeting follow. Those whose abstract numbers are followed by the letter *t* were read by title. Messrs. Botsford, Clifford, Martin, Matison, and Levine were introduced by Professor A. D. Michal.

1. *On the real multiplication of elliptic functions*, by Professor E. T. Bell. (Abstract No. 37–11–360.)

2. *On the integration of unbounded functions*, by Professor W. M. Whyburn. (Abstract No. 37–11–361.)

3. *On the distribution of roots of polynomials with integer coefficients*, by Dr. D. H. Lehmer (National Research Fellow). (Abstract No. 37–11–362.)

4. *On certain cyclotomic functions*, by Dr. D. H. Lehmer (National Research Fellow). (Abstract No. 37–11–363–*t*.)

5. *Contributions to the theory of multiple-point integral invariants*, by Mr. J. L. Botsford. (Abstract No. 37–11–364.)

6. *Continuity and differentials of implicit functionals*, by Mr. A. H. Clifford, (Abstract No. 37–11–365.)

7. *Concerning certain root location theorems*, by Professor Raymond Garver. (Abstract No. 37–11–366.)

8. *The explicit determination of the most general linear continuous functional transformation preservative of continuity*, by Mr. R. S. Martin. (Abstract No. 37–11–367.)

9. *A theorem of the limits of Stieltjes integrals*, by Mr. R. S. Martin. (Abstract No. 37–11–368–*t*.)

10. *Representation of function valued polynomial functionals of a continuous function*, by Mr. R. S. Martin. (Abstract No. 37–11–369–*t*.)

11. *Function-valued analytic functionals of several functions*, by Mr. R. S. Martin. (Abstract No. 37–11–370–*t*.)

12. *Contribution to the theory of analytic functions in abstract spaces* (preliminary report), by Mr. R. S. Martin. (Abstract No. 37–11–371–*t*.)

13. *Integral addition theorems for Bessel functions*, by Mr. Harry Matison and Mr. A. H. Levine. (Abstract No. 37–11–372.)

14. *Parallel displacement of a multiple-point tensor along several curves in Riemannian geometry*, by Professor A. D. Michal. (Abstract No. 37–11–373–*t*.)

15. *Relations between the projective and metric differential geometries of surfaces*, by Professor O. W. Albert. (Abstract No. 37–11–374.)

16. *An overlooked numeral system of antiquity*, by Professor A. A. Shaw. (Abstract No. 37–11–375.)

17. *A transformation of the lines of space effected by means of two quadratic reguli*, by Professor A. R. Williams. (Abstract No. 37–11–376–*t*.)

18. *Enumerative properties of r-space curves*, by Professor B. C. Wong. (Abstract No. 37–11–377–*t*.)

19. *On the application of a theta formula to representation in binary quadratic forms*, by Dr. Gordon Pall. (Abstract No. 38–1–3–*t*.)

T. M. PUTNAM,
Associate Secretary

THE N. R. C. REPORT ON ALGEBRAIC GEOMETRY

Selected Topics in Algebraic Geometry. Bulletin of the National Research Council, Number 63. Report of the Committee on Rational Transformations: Virgil Snyder (Chairman), A. B. Coble, Arnold Emch, Solomon Lefschetz, F. R. Sharpe, C. H. Sisam. Published by the National Research Council of the National Academy of Sciences, Washington, D. C., 1928. 395 pp.

The Division of Physical Sciences of the National Research Council in 1925 appointed a committee of six mathematicians, consisting of Virgil Snyder (Chairman), A. B. Coble, Arnold Emch, Solomon Lefschetz, F. R. Sharpe and C. H. Sisam, to draw up a report on the work to date in the field of rational transformations. The committee, after several meetings and a great amount of work both individually and collectively, submitted this report. As stated in the preface, although the main theme is the rational transformation, the committee did not confine itself strictly to this field, but considered broader topics involving or allied to geometric transformations both rational and irrational in the plane, three-space, and hyperspace. This broader treatment makes the report still more valuable.

The report consists of seventeen chapters written individually by the various members of the committee. Each chapter, complete in itself, gives a brief development of its topic, citing references to authors throughout. The references are arranged alphabetically according to the name of the author and are numbered serially. The complete list giving the author and periodical reference for each citation is found at the end of each chapter. The chapters are subdivided into sections and the section in which the reference is cited is given in a bracket at the right of the reference in the list at the end of the chapter for all chapters except 1, 2, 10, 14.

In the report, 2794 papers written by 1306 different authors are cited. References to these papers are skilfully woven into the discussion with concise appraisals of the contribution of the paper to the development of the subject. The report would be of great value if it were merely a bibliography of this size, but the pithy description of the content of the paper cited, together with the cross reference scheme described above enabling one who knows the name of the author to find the description of the paper in the report, increases its value many fold. These brief descriptions come from original sources and are therefore much more dependable than if obtained from synopses.

This comprehensive report is particularly pleasing in that so few essential typographical errors occur. This is chiefly due to the great care taken by the authors, and especially by the Chairman, in preparing the manuscript and reading proof. Practically all of the typographical errors that do occur are minor errors in punctuation, spelling of names and foreign words, size and style of type. The reviewer has not found one that would be really confusing to a careful reader.

In lines 8, 9, page 84, it is stated that Montesano proved (1905) that there exist only four symmetric types of plane Cremona transformations, namely

those of order 2, 5, 8, 17. This was previously established by Bertini* in 1877 in a footnote of the page cited. The first line of the table given in the footnote is incorrect. The equations (8), (9), page 252 from which the table is derived are, however, correct. Moreover, the correction in the solution was evidently made by Bertini that same year inasmuch as another table giving the characteristics of these four symmetric types correctly occurs as an erratum at the bottom of the second index page of the same volume.

In addition to being a descriptive bibliography, this report is a series of concise treatises covering the most important phases of algebraic geometry. It is indispensable to all research workers in that field.

Mathematicians are deeply indebted to the Chairman and members of this committee for their unselfish devotion to the preparation of this able report.

A brief description of the contribution of each member of the committee follows.

VIRGIL SNYDER: CHAPTERS 5, 6, 7, 9, 11.

Chapter 5. *Multiple correspondences between two planes.* The properties of (1, 2) point correspondences (or plane involutions of order two) and the birationally independent types of these transformations are derived. The four involution types of Cremona transformations, treated fully in Chapter 4, are briefly described here since the two image points in the simple plane are in (1, 1) correspondence, giving rise to a Cremona involution associated with each involution of order two. Involutions of general order are next discussed, followed by cyclic involutions of general period and (m, n) point correspondences. In this chapter, 127 papers are cited.

Chapter 6. *Involutions on rational curves.* The geometry on a rational curve is treated here only from the standpoint of associated linear involutions. After discussing involutions on the line and conic, a section is devoted to the general properties of an involution of order n without regard to carrier. Following this, involutions on rational plane curves and rational space cubics and quartics are treated and finally involutions on rational carriers in hyperspace. This chapter cites 253 papers.

Chapter 7. *Correspondences on non-rational curves.* Chasles' principle of correspondence for rational curves is discussed as to history, applications, and extensions, notably the extension by Cayley and Brill to non-rational curves. Zeuthen's formula relating the characteristics of a multiple correspondence between two curves is given, followed by applications of correspondences to cubics and quartics and a treatment of the Brill-Noether theory. In the discussion of the reduction of singularities, the author mentions his own paper establishing the existence of all algebraic plane curves with distinct nodes. It should be said that this paper marks the first successful attack on the problem of the existence of algebraic curves with assigned singularities. The chapter closes with a discussion of irrational involutions and the relations of involutions to enumerative geometry. This chapter cites 434 papers.

Chapter 9. *Involutions and* (1, 2) *correspondences in* S_2, S_2'. Beginning with

* *Richerche sulle transformazioni nel piano*, Annali di Matematica, (2), vol. 8 (1877), p. 271.

the simplest case, that of reciprocal radii, involutorial transformations of space are discussed. A full treatment of (1, 2) correspondences is given, followed by involutions of lines and systems of lines associated with (l, m) correspondences. Some interesting extensions and exceptions in four dimensions are given. The chapter cites 160 papers.

Chapter 11. *Multiple correspondence and mapping in space and hyperspace.* The chapter opens with a discussion of particular cases of (l, m) point correspondences in S_2. The mapping of the lines of S_3 satisfying one or no conditions on the points of S_3 or S_4 respectively and of linear line complexes on other elements are treated in some detail. Multiple correspondences in hyperspace are then discussed and finally compound involutions, that is, (m, n) correspondences composed of two involutions of orders m and n such that the two image points in the multiple planes are connected by a Cremona transformation. The chapter cites 186 papers.

ARTHUR B. COBLE: CHAPTERS 4, 8.

Chapter 4. *Planar Cremona transformations.* The first four sections of this chapter deal with properties of Cremona transformations when the two corresponding planes are not necessarily superposed. The general properties of Cremona transformations are developed and these transformations are then classified. Linear transformations associated with Cremona transformations, products and groups of Cremona transformations are discussed together with their geometric interpretations. In sections 5–10, the corresponding planes are superposed. Fixed points and curves and cyclic sets of points are discussed briefly. Involutorial Cremona transformations are next treated quite fully beginning with Bertini's four birationally independent types and proceeding to other plans of classification and methods of construction. Descriptive references are given to many proofs of the fundamental theorem that every Cremona transformation can be expressed as a product of quadratic transformations. Closely associated with this theorem are many theorems, general and particular, concerning the birational equivalence of linear systems of curves. This subject is fully treated. Periodic transformations and groups of transformations are discussed and the chapter closes with some applications of Cremona transformations to geometry and algebra. The chapter contains 320 citations.

Chapter 8. *Cremona transformations in space and hyperspace.* The description of space transformations follows the same order as for the plane. Involutorial space transformations are, however, omitted and are treated in Chapter 9. Regular groups of Cremona transformations in hyperspace are discussed. Also at the end of some of the sections, the results are extended to hyperspace. Many of the properties of Cremona transformations of higher space are similar to those of the plane. The most important different feature for S_3 is that a general Cremona space transformation can not be expressed as a product of a finite number of Cremona space transformations of lower order. The chapter cites 224 papers.

ARNOLD EMCH: CHAPTERS 1, 2, 10.

Chapter 1. *Quadratic Cremona transformations.* Quadratic Cremona transformations are first treated from the historical point of view beginning with the

first known examples and concluding with the discovery of the general form of such transformations by Cremona in 1862. The properties of general quadratic transformations are developed synthetically and algebraically, after which follows a discussion of special forms of these transformations, including circular inversion, and geometric applications. Groups of transformations are now treated and finally systems of linear transformations associated with quadratic Cremona transformations. The chapter cites 265 papers.

Chapter 2. *Analysis of singularities of plane algebraic curves.* The historical treatment follows the plan of Enriques in the second volume of *Lezioni sulla Teoria Geometrica delle Equazioni e delle Funzioni Algebriche* and deals with the development of the theory of algebraic singularities from many points of view. It also includes a short discussion of the still largely unsolved problem of the existence of algebraic curves with assigned singularities. The analysis of singularities is treated chronologically. The projection of a singular plane curve into a non-singular hyperspace curve is discussed. The chapter closes with an application of the method of resolving a singularity to determining the number of intersections of two curves with given singularities at a given point. In the chapter, 171 papers are cited.

Chapter 10. *Reduction of singularities of space curves and surfaces.* In a very brief chapter, the author treats developments in this subject since 1915, referring the reader to accounts by Castelnuovo, Enriques, and Berzolari for earlier contributions. Seventeen papers written since 1915 are cited. There is also a list of earlier works containing 41 items. The titles of these papers are given but they are not discussed.

SOLOMON LEFSCHETZ: CHAPTERS 15, 16, 17.

Chapter 15. *Transcendental theory.* The chapter opens with a discussion of Riemann surfaces and abelian integrals associated with an algebraic curve. Sections 2–5 deal with a non-singular, irreducible surface F in any space as follows: Linear systems of curves on F and the properties of F associated with them; topological properties of F; generalized abelian integrals associated with F; the distribution of curves on F into discrete continuous sets of linear systems. Extensions of some of the above results to varieties in hyperspace are briefly treated. The chapter concludes with a glossary of terms and notations used in this and the two succeeding chapters. Sixty-two papers are cited. A list of recent books is also given.

Chapter 16. *Singular correspondences between algebraic curves.* Algebraic correspondences between the points of the same or distinct curves are singular if they impose restrictions on the Riemann matrices of the curves, otherwise non-singular. Non-singular correspondences are treated in Chapter 7. Singular correspondences between points on the same curve are developed and discussed from several points of view, and this is followed by a short section on the closely allied theory of correspondences between the points of two distinct curves. The next problem dealt with is that of determining all correspondences between the points of a fixed and a variable curve, and the inverse problem, which latter involves the construction of regular Riemann surfaces. The reduction of abelian integrals and its relation to the determination of irrational involutions is treated

briefly. Irrational series of points on a curve and their relations to correspondences between two curves are investigated. The chapter closes with a discussion of birational transformations of points on a non-rational curve. One hundred nine papers are cited.

Chapter 17. *Hyperelliptic surfaces and abelian varieties.* First there is given a condensed but comprehensive review of the basic definitions and properties of multiply periodic and related functions, closing with the definition of an abelian variety. The study of hyperelliptic surfaces is chronological. The subject is fully treated and the important investigations described with clarity and some detail. The general properties of abelian varieties are developed. A treatment of impure matrices and the varieties associated with them follows. The relation of geometric transformations to complex multiplication is then shown, and this is followed by a discussion of complex multiplication. One hundred forty-nine papers are cited.

FRANCIS R. SHARPE: CHAPTERS 3, 12, 13.

Chapter 3. *Linear systems of plane curves.* In three pages, the author gives a comprehensive treatment of linear systems of plane curves including a discussion of superabundant systems, adjoints, and composite systems. Twenty-one papers are cited.

Chapter 12. *The mapping of a rational surface on a plane.* Methods of mapping all rational three-space surfaces of orders less than or equal to five are given and those cases which have been investigated for orders greater than five, including the general order n. The general theory of mapping a rational surface on a plane is developed, and this is followed by discussions of the projection of surfaces of hyperspace into surfaces of S_3, the relation of rational surfaces having plane sections of given genus to linear systems of curves of least order and that genus, rational surfaces that may be mapped on multiple planes, general conditions for a surface to be rational, and some special rational surfaces. One hundred forty-seven citations are made.

Chapter 13. *The mapping of a rational congruence on a plane.* This brief chapter deals with the mapping on a plane of congruences with a finite or a singly infinite number of singular points, closing with a discussion of congruences defined by quadrics and projectivities. Twenty-eight papers are cited.

CHARLES H. SISAM: CHAPTER 14.

Chapter 14. *Involutions on irrational surfaces.* After defining an involution of order n between the points of two algebraic surfaces, the author classifies such involutions according as the number of their coincidences is infinite or finite. The general characteristics of involutions of each of these two types are developed and some interesting special cases of each type discussed. Seventy-three papers are cited.

T. R. HOLLCROFT

SHORTER NOTICES

Electricity and Magnetism. By Vincent C. Poor. New York, Wiley, 1931. vii+183 pp.

This book is divided into five chapters having the titles Vector Analysis, Electrostatics, Magnetism and Polarized Dielectrics, Current Electricity and The Dynamics of the Electric Current, The Electron Theory and The Application of Special Relativity Theory to Electricity. The fact that only eight pages are devoted to vector algebra and vector calculus and that the entire text consists of one hundred seventy-five pages, gives an idea of the compressed character of the treatment of this vast subject. In his preface the author says "This monograph is in no way intended as a treatise, but rather as a *path through* the subject of electrical theory. According to the observation of one of the referees, it should be a 'big time-saver.'"

It seems to me that the path is not very clear and that it is questionable whether the degree of conciseness is conducive to saving time. Had the book been expanded judiciously to double its size, without introducing new topics, it might have become even a bigger time-saver. In any case the object of a book on such a difficult subject should be, it seems to me, to give the student a clear, comprehensive grasp of the subject and not merely to save time.

In spite of the introduction of the electron theory in the last chapter, the book is written from the point of view which antedates the electron theory. This explains the section headings, "Fictitious Magnetism" and "Fictitious Electricity." It would have been better to present the subject in the light of the progress of electrical theory during the last thirty-five years and to present the subject throughout from the point of view of the electron theory. This would have made it unnecessary to call real electrical phenomena at the boundaries of dielectrics "fictitious."

The notation of the book is at times confusing. For example, "curl" and "rot" are used indiscriminately even on the same page; on the other hand "divs" and "curls" are used where "div" and "curl" would have done just as well.

The Lorentz transformations are introduced on the hypothesis of the Fitzgerald-Lorentz contraction and later it is pointed out that these satisfy the special theory of relativity. The derivation of these transformations from the postulates of relativity, which can be done very simply, would have carried a greater conviction of their reality to the student.

The author rightly states in his preface "In general, the professional electrical engineer is graduated without being able to read electrical theory. This is a deplorable situation; the theory of electricity should not be a closed book to the good electrical engineer." His remedy, however, given in the statement "It would seem feasible to correct this state of affairs by a one-hour course for junior and senior electrical students," is not adequate. It seems to the present reviewer that most of the shortcomings of the book are due to this feeling on the part of its author that the theory of electricity can be adequately covered in a very short course.

H. M. Dadourian

Leçons sur les Conduites. By Charles Camichel. Paris, Gauthier-Villars, 1930. iii+101 pp.

This book on hydraulics is devoted to the discussion of the propagation of pressure waves in fluids in pipe-lines of constant and varying thickness. A minimum of the mathematical theory of hydrodynamics is assumed and the emphasis is placed on the industrial applications.

F. D. MURNAGHAN

Counting and Measuring. By H. von Helmholtz. Translated by Charlotte Lowe Bryan, with an Introduction and Notes by Harold T. Davis. New York, Van Nostrand, 1930. xxxiv+39 pp. $2.50.

This is a translation of one of Helmholtz's well known contributions to the foundations of science. The question considered is: How can the concept of number be logically employed for the purpose of comparing physical magnitudes, and what attribute must the combination of magnitudes possess in order that it can logically be correlated with the addition of numbers? The concept of number itself is first examined and is made to depend on an intuitive time-sequence. The numbers so defined are of course the ordinals; the cardinals follow immediately by an appeal to the empirical process of counting. Both types of number are then shown to satisfy the axioms of arithmetic. Perhaps the most profound idea in this work is that the relation of equality between physical magnitudes satisfies the necessary axioms (and thereby is expressible as an equality between numbers) not on account of any law of nature; it is rather that the axioms determine what physical relations are to be regarded as relations of equality. Similarly the physical combinations which are to be regarded as addition are only those which satisfy the axioms of addition.

It is of course easy to criticize Helmholtz's treatment of number in the light of modern mathematical logic and his treatment of mensuration in the light of modern physics. The significance of the present work seems therefore to be chiefly historical. In the introduction, which is entitled "A Survey of the Problem of mathematical Truth," Professor Davis says of this work that it " . . . stands historically in the midst of a stream of speculation which today has progressed far beyond its original mathematical bounds and has become an active force in some of the most fundamental thought of modern physics." This is Professor Davis's only reference in his survey to the work in question. He traces rather the course of this stream of speculation which started with the paradox of Zeno and has finally led to the Hilbert-Brouwer controversy, the "postulate of boundedness" of Eddington, and the "principle of indeterminacy" of Heisenberg. But in spite of recent intellectual floods, one gathers from the last sentence of the survey, which contains Poincaré's scornful remark about logistics, that the problem of mathematical truth remains unsolved. One wonders why Professor Davis did not comment further on Helmholtz's ideas and more completely evaluate the work to which the book is devoted. One will agree, however, that this work is in many ways a masterpiece and thoroughly worth translating. It is the work of a great physicist who saw the need of scrutinizing critically the logical foundations of his science; the importance of such examination is being realized today as never before.

P. A. SMITH

Conférences d'Actualités Scientifiques et Industrielles. Année 1929. With a preface by J. Lemoine. Paris, Hermann, 1930. viii+270 pp.

This book contains a series of lectures, independent of each other, given by various scientists at the *Conservatoire national des arts et métiers* during the year 1929. This is the third such series, the first having been given in 1927. Three of the lectures have a particular appeal to those interested in recent developments in theoretical physics. One of these is by L. de Broglie on *Une crise récente de l'optique ondulatoire.* This fine discussion of the foundations of wave mechanics by the genius who created this theory closes with the pregnant remark, "Nous saurions bien des choses si nous savions seulement ce qu'est un rayon de lumière."

The other lectures referred to are *Les atomes de lumière et les quanta* by E. Bloch and *Les problèmes de structure et la mécanique nouvelle* by L. Bloch. All the lectures are non-technical, the idea evidently being to present the essential features without the distraction involved in details.

F. D. MURNAGHAN

Fonctions Automorphes. By Pierre Fatou. (Volume II of Appell and Goursat's *Théorie des Fonctions Algébriques.*) Paris, Gauthier-Villars, 1930. xiv+521 pp.

Some years before his death, which occurred in 1929, the brilliant analyst P. Fatou undertook the preparation of a revised edition of Appell and Goursat's *Fonctions Algébriques.* Fatou was invited by Professor Goursat to add to the original edition a treatment of automorphic functions. He responded by writing a comprehensive exposition of that subject, which it was found desirable to publish in a separate volume. Thus, Chapters 13 to 16 of the *Fonctions Algébriques,* which make up the second volume of that work, really form an independent treatise, due entirely to Fatou.

Chapter 13 begins with a study of the elementary properties of linear transformations of a complex variable z. With a free use of projective geometry, a knowledge of which is assumed, groups of linear transformations of z are shown to be isomorphic with groups of motions in non-euclidean spaces. Fatou uses Montel's theory of normal families of functions to obtain a new and very simple proof of the theorem that a group of real linear transformations which has no infinitesimal transformation is properly discontinuous. It is also proved that every group which has no infinitesimal transformation induces a properly discontinuous group of transformations of three-dimensional space.

Chapter 14 deals with the determination of the fundamental domains of Fuchsian and of Kleinian groups, and with the construction of such groups through the stipulation, in advance, of their fundamental domains. Chapter 15 presents the chief known properties of Fuchsian and of Kleinian functions. The theta series are studied in detail. Chapter 16 gives an interesting account of conformal mapping and uniformization.

Fatou's style is rapid, but clear. One sees that he insisted everywhere on complete rigor, and that he refrained from presenting conjectures as facts. His masterly treatise will sustain, and encourage the advancement of, the elegant theory of Schwarz, Schottky, Klein, and Poincaré.

J. F. RITT

Von Zahlen und Figuren. By Hans Rademacher und Otto Toeplitz. Berlin, Springer, 1930. 6+164 pp.

The book under review is written for the intelligent man in the street and not for mathematicians. It presupposes very little of mathematical background although it describes a number of difficult problems, some of which are unsolved to this day.

The authors discuss 22 problems chosen from the fields of number theory, geometry, and point-set theory. The subject matter is rather cleverly arranged; for example, the four-color problem is followed by that of Fermat. It results that in the normal process of reading the book, one does not become tired of the sameness of the subject.

The treatment of all problems is very clear, the emphasis being placed on the logic and reasoning involved in the proof. Many people unfamiliar with mathematics have an idea that it is a collection of tricks for rapid calculation or the solution of involved numerical problems; a perusal of this book shows clearly how erroneous this idea is.

The book makes very pleasant reading, containing, as it does, not only interesting problems but also a bit of their history. In the opinion of the reviewer every teacher of elementary mathematics would do well to get acquainted with this delightful little book.

M. S. KNEBELMAN

Partielle Differentialgleichungen der mathematischen Physik. By A. G. Webster and G. Szegö. Leipzig, Teubner, 1930. 528 pp.

Since A. G. Webster's *Partial Differential Equations of Mathematical Physics*, edited by Samuel J. Plimpton, has been on the market since 1927, a brief note should suffice to characterize the new edition. The original publication, in English, is a comprehensive treatment of classical wave equations, or more specifically, a treatment of methods of solution of boundary value problems, in particular, the vibration of strings, pipes, membranes, rods, thin plates, also the linear flow of heat, and classical electromagnetic waves. The questions treated are those of classical physics. The partial differential equations of modern physics, the Schrödinger and Dirac equations, as one would expect, are not considered.

The German edition is somewhat more than a mere translation of the original one, though it carries the same article number throughout. The subject matter treated is exactly the same, though the process of revision has augmented the work by 88 pages, making a 528 page book of it. This added material includes an index of ten pages, while the rest of the material is distributed largely through the various articles of Chapters IV, V, VII, and VIII, as is stated in the preface. The result is that numerous articles have been subjected to a more rigorous mathematical treatment than is presented in the English edition.

The revision, like the English edition, is extremely rich in content, but contains no modern physics. For many purposes the English edition will be found very satisfactory; the chief advantage of the new book lies in the more rigorous mathematical presentation of portions of the subject matter.

V. C. POOR

Gesammelte mathematische Werke. By Richard Dedekind. Zweiter Band. Herausgegeben von Robert Fricke, Emmy Noether, Oystein Ore. Braunschweig, Vieweg, 1931. 2+442 pp.

The first volume of Dedekind's collected works was noticed in this Bulletin (vol. 36 (1930), pp. 611–612). The second volume maintains the high standard of editorial excellence achieved in the first. As detailed summaries of the contents of this volume are readily available in the recent German notices, it will suffice here to call attention to one feature not found in the previous volume: eleven papers, notes, or letters from the "Nachlass" are here printed. These are of great interest, for historical reasons, if no other, and alone demand that the present volume be in every mathematical library. The letters to Frobenius are of special interest, as they give a glimpse into Dedekind's methods of work.

E. T. BELL

Versicherungswesen, System der Versicherungswirtschaft. By Alfred Manes. Leipzig and Berlin, Teubner, 1931. x+315 pp.

The book brings together in one volume a treatment of various types of property and business insurance. The discussion is concerned with much that has appeared in American practice under the name of casualty insurance together with a treatment of fire and marine insurance. The book contains no formal mathematical treatment of any branch of insurance, but gives a general discussion of the development of underwriting ideas in the various branches.

H. L. RIETZ

Die Anwendung der Nomographie in der Mathematik. By H. Schwerdt. Berlin, Springer, 1931. 8+116 pp.+104 tables.

The applications of nomography seem, in general, to have been put forth with the intention of relieving the non-mathematician of the labor of computing, and of the necessity of thinking. However, the volume under review is addressed to mathematicians. Although not trivial, the text makes no great demands on the knowledge or ability of the mathematically inclined reader, and his interest is certain to be aroused by the clever application of the principles of nomography to the formulas so familiar to him. The range of illustration is large. In five chapters on elementary geometry, coordinate systems, coordinate geometry, equations, and functions, the author gives over two hundred varied applications, illustrated by excellent drawings in the collected Tables. The importance and usefulness of these applications vary. Thus a nomogram for $a = 2r \sin \alpha$ cannot give the degree of accuracy usually desired, and as a rough check is inferior to a slide rule or a mental calculation. Nomograms for the roots of cubic equations are useful. We found of most interest the sections on space coordinate systems, on curvatures of surfaces, and on the solutions of equations. The book may be assigned to a good undergraduate for independent study.

There is a bibliography and a good index. The proof reading seems to have been done with unusual care, and the paper, printing, and binding are of unusual excellence.

B. H. BROWN

Quozienti e Resti nella Divisione di Frazioni. Congruenze a Modulo Frazionario. By Paolino Fulco. Memoria IV. Civatavecchia, 1929. 104 pp.

Dr. Fulco undertakes in minute detail a systematic exposition, by elementary methods, of the theory of linear and quadratic congruences with rational coefficients, in one unknown, to a rational modulus. The theory is based on the following definitions. The fraction a/b is divisible by m/n if an/bm is an integer; if a/b is not divisible by m/n, the incomplete quotient of a/b by m/n is the greatest integer in an/bm.

Misprints unfortunately are rather numerous; the second numerical example illustrating the second definition already contains one. Possibly it is on account of similar slips in printing that the reviewer is unable to follow some of the proofs. In particular, the stated necessity and sufficiency of the conditions for the solvability of quadratic fractional congruences seem to the reviewer to demand more detailed proof than the author indicates.

E. T. BELL

Vorlesungen über Allgemeine Mechanik. By A. Brill. Munich and Berlin, R. Oldenbourg, 1928. viii+356 pp.

This treatment of the dynamics of a particle, rigid bodies, and systems of particles is an outgrowth of Professor Brill's lectures on theoretical mechanics in the Technische Hochschule in Munich and the University of Tübingen. The statics of rigid bodies is discussed only incidentally, and for the mechanics of deformable bodies, with the exception of two sections on the elastic rod, the reader is referred to the author's *Einführung in die Mechanik raumerfüllender Massen* (Leipzig and Berlin, 1909). The subject proper, the dynamics of particles and rigid bodies, is built up from first principles in a manner which commends the book to the beginning student of mechanics who has mastered mathematics through advanced calculus.

Part I, the dynamics of a material point, begins with a thorough discussion of kinematics, including the introduction of the vector calculus. The author then devotes one chapter to central forces and another to the theory of the potential. This is followed by chapters on constrained motion, the problem of two or more bodies with applications to the theory of double stars, and a short chapter on relative motion. Part II, the rigid body, considers first the kinematics of rigid bodies, the conditions for equilibrium, and statics. The dynamics of systems of particles and rigid bodies is then discussed from a general point of view, beginning with d'Alembert's principle and the more immediate general laws. Their development is interrupted to allow applications to the theory of the gyroscope and is resumed in the next chapter with Lagrange's equations for general systems, including non-holonomic systems, and Hamilton's principle. The last chapter is devoted to the theory of impulsive forces.

Professor Brill's work is to be recommended as an exceptionally well written introduction to dynamics. It is particularly to be commended for the fine balance which it maintains between general principles on the one side and applications to physics, astronomy, and engineering on the other. One can only wish that such a book of similar compass were available in the English language.

H. P. ROBERTSON

Mathematische Statistik der Personenversicherung. By G. Rosmanith. Leipzig and Berlin, Teubner, 1930 (Sammlung Mathematisch-Physikalischer Lehrbücher—E. Treftz). vi+141 pp.

In his preface, the author states that he is trying to present in simple form material found largely in the books of such authors as Czuber, Blaschke, and Broggi. This has been done admirably—with the omission of difficult proofs—so that the book is within reach of students who have had a good one-year course in calculus.

The book does not pretend to be a text on actuarial mathematics. It does not carry the student through the calculation of premiums and reserves in life insurance. It takes up, rather, the processes connected with the construction of mortality tables, and explains in an interesting manner numerous methods used in graduation.

The first chapter deals with calculations that can be based upon the experience of insurance companies. Withdrawals are considered—the number supposed known—in the construction of *Select Tables.* Subnormal lives are discussed, as is also disability. In the second chapter population statistics are discussed, with graphical representations of the two survivor aggregates and the three death aggregates, from which the elementary death aggregates are to be computed. In Chapter 3, continuous variables are introduced, properties of the force of mortality are developed, also formulas for the probabilities of exit from a group through several causes, death, disability, etc. Then follow chapters dealing with the mortality laws proposed by De Moivre, Lambert, Gompertz, Makeham, Wittstein, Lazarus, Quiquet, and Dormoy. To the list given could have been added the product of the Makeham function by h^{x^2}. The computation of parameters for theoretical mortality curves is given in some detail, the King-Hardy method and the least square method with various adaptations. The last chapter deals with graduation by averaging and kindred processes, exhibiting the methods of Woolhouse, Karup, Jörgensen, Sprague, Higham, King, Gram, and Koeppler.

In a short appendix, the elements of the theory of probability are presented, followed by a sketch of Bernoulli's Theorem and its inverse, Bayes' Theorem, adjustment of measurements, Lexian ratio, frequency curves, and correlation. In places the appendix appears too condensed; in particular, on page 118, second paragraph; page 122, first two lines; page 134, second paragraph, this being an attempt to characterize in nine lines the Pearson system of curves. More light, however, is thrown by the example of a Pearson Type III, which follows. On page 127, the arithmetic mean is declared to be the "most probable" value—as is so often done. On page 132, the Lexian ratio is unfortunately inverted. At the middle of page 136, the range of the coefficient of correlation is set down as from 0 to 1, instead of from -1 to 1. Numerous numerical illustrations in the appendix make this part of the book decidedly helpful, in spite of its brevity.

Two mortality tables, the English Healthy Males HM, and Höckners LM, terminate the book

A few misprints have crept into numbered formulas, such as μ_x for p_x, page 57, line 1; $x+n$ for $x+1$, page 60 (24); t for $-t$, page 87, (3); an extra x_t in the

numerator of the last fraction, page 90, (11). On page 69, t appears in the denominator instead of i, and r is introduced without explanation. At the foot of page 61, it is not clear whether the probabilities are for exit or for remaining. At the foot of page 35, the nth root appears to be required. For the most part, the printing seems to be very good.

Rosmanith's book would be of great value to an actuary, as it gives in compact form so many different methods for constructing tables upon which the superstructure of actuarial mathematics is built.

E. L. DODD

Bibliography of Projective Differential Geometry. By Pauline Sperry. University of California Publications in Mathematics (Vol. 2, No. 6 (1931), pp. 119–127).

It has been said that a mathematician's laboratory is his library. The author of this remark neglected to tell us whether a bibliography of a particular field of mathematics is a test tube. At any rate a reliable bibliography is an essential instrument.

In the field of projective differential geometry anything like a complete bibliography has been lacking until recently. The reviewer in the current volume of this Bulletin called attention to an extensive and very valuable bibliography in the book *Introduction à la Géométrie Projective Différentielle des Surfaces* by Fubini and Čech.

Now we have to thank Miss Sperry for a bibliography of the publications of the projective differential geometers of the United States, Canada, and Japan. This bibliography has been compiled with painstaking care, and is complete as far as it goes. Actual count indicates that it includes references to the works of twelve authors and to sixty-one papers omitted from the bibliography of Fubini and Čech.

E. P. LANE

Lehrbuch der Funktionentheorie. By L. Bieberbach. Vol. I, third edition. Leipzig and Berlin, Teubner, 1930. vii+322 pp.

This revised edition is not appreciably an enlargement. The reviewer noted a few additions to the first edition: Schwarz's inequality, a theorem on power series, and some historical matter on normal families. Various errors in the earlier edition are eliminated, certain changes in notation are made, and numerous slight textual alterations improve the presentation. The most important change is the introduction into the text of sub-headings. These are in heavy type. They improve the appearance of the page and make the finding of desired subjects a great deal easier.

The third edition is thus not essentially different from the first edition of ten years ago. Bieberbach's volume remains, however, one of the best introductions to the theory of functions of a complex variable. It is comprehensive enough for the purpose, and its presentation is clear. The development leans less on the real variable than that of most texts. The point of view is thoroughly modern. The author has applied his unusual skill as a text-book writer to his favorite field with very happy results.

L. R. FORD

Erreurs et Moindres Carrés. By R. Deltheil. Paris, Gauthier-Villars, 1930. v+161 pp.

This treatise is the second paper of an excellent series of monographs on probability, entitled *Traité du Calcul des Probabilités et de ses Applications*, edited by Émile Borel, with the collaboration of L. Blaringhem, C. V. L. Charlier, R. Deltheil, H. Galbrun, J. Haag, R. Lagrange, F. Perrin, and P. Traynard.

On the theoretical side, Deltheil sketches in a masterly way two of the main routes rigorously leading to the so-called Gaussian probability law (Chapters 2, 4, 6, 7). The first of these is the method of moments, using staircase functions, going back to Tchebychef, Bienaymé, Stieltjes, Castelnuovo, and Liapounoff. Deltheil refers the readers to the monograph written by Borel as the first paper of the *Traité* for further details. The second rigorous method is that of Paul Lévy who uses as a "characteristic function" the Stieltjes integral of $e^{ixt}dF(x)$, where $F(x)$ is the cumulative frequency law. This characteristic function goes back to Cauchy, and still further back to Laplace.* Deltheil's treatment here is a felicitous condensation of Lévy's excellent *Calcul des Probabilités*, to which the reader is referred for details. An interesting discussion of the hypotheses required by the two methods of approach to the Gaussian Law is given (pp. 80–82), in which some later work by Fréchet † is also mentioned, whose hypotheses lead to a different law.

In any comprehensive treatment of least squares, some use or at least mention of Bayes' Theorem seems inevitable, in spite of the difficulties involved in making the treatment rigorous. So long as the a priori probability is under control, no special obstacle arises; and Deltheil's Chapter 1 gives a very satisfactory and instructive treatment for this aspect. Chapter 3 picks up Bayes' Theorem again for use with measurements where, in general, the a priori probabilities are not known and cannot be known. The hypothesis of constant a priori probability is accepted temporarily as the only reasonable assumption in case of total ignorance, but the reader is referred to Chapter 9 for release from this objectionable assumption. In Chapter 5, devoted to the principle of the arithmetic mean, the difficulty reaches its culmination. The original "demonstration" of Gauss is given, and some of the numerous objections raised thereto. Deltheil concludes (p. 63) that the critics of Gauss are justified. Poincaré's attempt is given (pp. 59–63) to salvage something from the wreck. Here, the principle of the arithmetic mean as the most probable value leads to the assumption that the a priori probability $\phi(x)$ is a constant. A rather peculiar constant, indeed, this would be; for the integral of $\phi(x)$ from $-\infty$ to $+\infty$ must be unity. Possibly, this "proof" of Gauss, abandoned by Gauss himself (p. 63), has great "historical value"; since nearly every book on least squares incorporates it—often reluctantly. But, perhaps the space devoted to such obsolete material could be better used. Instead of clinging to the untenable "$\phi(x)$ = constant," it is possible to show that under very broad conditions the effect of $\phi(x)$

* E. C. Molina, *The theory of probability: some comments on Laplace's Théorie Analytique*, this Bulletin, vol. 36 (1930), pp. 369–392.

† *Sur l'hypothèse de l'additivité des erreurs partielles*, Bulletin des Sciences Mathématiques, 1928.

is evanescent as the number of measurements increases. One of the most comprehensive and far-reaching treatments along this line is that of R. von Mises,* whose results, expressible with k-dimensional vectors, are of great generality.

Deltheil does to some extent disentangle his treatment from Bayes' Theorem in subsequent chapters. Error-risk is introduced and *linear* functions of the measurements (p. 101) are considered. It is, indeed, possible to prove rigorously that under the Gaussian Law the arithmetic mean has a probability greater than that of any other linear function or *weighted mean*, in case the "precision" is constant, and to prove the analogous theorem for measurements of unequal precision. So long as we make comparisons only among linear functions, we can move on safe ground.

The subject of least squares, as a practical tool for adjustment of measurements in geodesy and kindred sciences, has a rather well-defined content. Deltheil treats in an admirable manner the topics generally required, giving numerical illustrations in considerable detail. He also sketches a few other topics such as the Gram-Charlier development in Hermite polynomials (pp. 87–90) and Poincaré's method of successive approximations (pp. 94–96). A five-place table of the probability integral concludes the volume.

E. L. DODD

Methoden der Mathematischen Physik. Bd. I, zweite verbesserte Auflage. By R. Courant and D. Hilbert. Berlin, Springer, 1931. xiv+469 pp.

The first edition (1924) of this important and useful book has been already reviewed in a very detailed manner by E. Hille (this Bulletin, vol. 31 (1925), pp. 456–459) and besides is so well known that we may restrict ourselves here to a few remarks. Although the number of pages has not increased considerably (xiii+450 in the first edition) the number of changes is large. Practically all the chapters and sections contain modifications, in exposition as well as in the order of the material. Among the most important additions the following should be mentioned separately: proof of the completeness of the sets of Laguerre's and Hermite's polynomials (Chapter II, 9.6); transformations of problems of the calculus of variations (Chapter IV, 9) where, on the basis of a recent paper by K. Friedrichs (Göttinger Nachrichten, 1929, pp. 13–29) it is shown that in many cases a minimum problem can be transformed into an equivalent maximum problem, with the same value of the extremum in question. The occurrence of a continuous spectrum in problems of mathematical physics is illustrated, of course, by the example of Schrödinger's equation (Chapter V, 12.4). In Chapter VI, 5, a generalized Schrödinger equation is treated from the point of view of the calculus of variations. However, no satisfactory treatment of the continuous spectrum is obtained in this fashion.

The bibliographical references are a little more complete in the present edition than in the first one. In this connection the reference to an unpublished paper by R. G. D. Richardson should be welcomed (p. 404). This paper was

* *Fundamentalsätze der Wahrscheinlichkeitsrechnung*, Mathematische Zeitschrift, vol. 4 (1919), pp. 1–97.

submitted to the editors of the Mathematische Annalen shortly before the World War. It contained numerous points of contact with the results of Chapter VI (Dependence of the characteristic values of elliptic boundary value problems on the domain and on the coefficients of the problem, zeros of the fundamental functions and the like). A reference to a recent paper by S. A. Janczewsky (Annals of Mathematics, vol. 29 (1928), pp. 521–542, particularly p. 542) would not be out of place on page 393 in connection with the corrected statement of a property of the nodes of the fundamental functions. It was Janczewsky who pointed out that the corresponding statement on page 365 of the first edition was not correct.

Some of the mistakes of the first edition are corrected in the present one, while some others are not. It is a little disappointing, for instance, that the erroneous treatment of zeroes of Bessel's functions (pp. 427–428) and an inconsistent definition of the Legendre's functions of the second kind (p. 436), which were pointed out by the reviewer of the first edition, are preserved in the second edition. A certain elegant vagueness of the style, which was also noticed by the reviewer of the first edition, of course, is preserved in the second one. It is desirable, however, that the statements of some results be made more precise. For instance, the statement of Picard's necessary and sufficient conditions for the existence of a solution of a Fredholm integral equation of the first kind is not correct as it stands on pages 135–136. The boundedness of the integrals $\int K(s, t)^2 ds$, $\int K(s, t)^2 dt$ on page 129 is not needed for the possibility of extension of the classical results of Fredholm theory to unbounded kernels. The statement concerning the sequences converging in mean (p. 93), which was erroneous in the first edition (pp. 96–97), is now correct, but fails to emphasize the essential fact that the function $f(x)$ to which the sequence converges in mean is uniquely determined, and that the whole property is merely a modification of Cauchy's criterion for the existence of a limit of a sequence, and states in essence the completeness of Hilbert's space.

The present reviewer shares with his predecessor the eager expectation (not yet realized) of the promised Volume II of the treatise. A quantity of most important facts concerning the existence of solutions and the precise formulation of the conditions under which these solutions exist are referred to the second volume. Without this volume at hand, the reader can not help the feeling of a certain lack of a solid foundation for many theories developed in the first volume. Hence, thankful as we are for the completion of the present second edition of the first volume of this excellent treatise, we have to agree with one of the authors of the book (R. Courant): "Das Erscheinen dieser zweiten Auflage legt mir mit verdoppelter Stärke die Verpflichtung auf, den zweiten Band, mit dem zusammen dieser vorliegende erst ein abgeschlossenes Ganzes bilden wird, in Druck zu geben."

J. D. TAMARKIN

Algebraische Theorie der Körper. By Ernst Steinitz. Edited by Reinhold Baer and Helmut Hasse. Berlin and Leipzig, de Gruyter, 1930. 150+27 pp.

The present book is a separate edition of Steinitz' well known paper on abstract fields, which appeared in the Journal für Mathematik (vol. 137 (1910)). Steinitz's paper has given rise to numerous investigations on abstract fields and

rings and it will always be considered as one of the classics of this branch of mathematics. I shall not give any account of the contents of this paper since I have already in a recent address (this Bulletin, August, 1931, discussed Steinitz's results and some of their consequences.

It is of interest to observe that Steinitz had but few precursors in these investigations, in spite of the fundamental character of his paper; only the ideas of Dedekind from the supplements to the fourth edition of Dirichlet-Dedekind's *Number Theory* have a direct bearing on Steinitz' problem, and they must have been particularly useful in the investigations on adjunctions of finite rank. The problem to determine all fields in which the Galois theory of equations is valid seems first to have been proposed by H. Weber.

Steinitz's paper is for the greater part easily read; only the chapter on Galois fields, particularly where the Zermelo axiom is used, seems unnecessarily complicated. To remedy this the two editors have added an appendix of explanations and notes and some of the most complicated proofs have been simplified. For those students who want to use the book as an introduction to abstract algebra these notes will be a valuable assistance; if I should venture a criticism it might be that they are in a few places a little too obvious. Very good presentations of the ideas of Steinitz can also be found in Haupt's *Algebra* (Leipzig, 1929) and in the first volume of van der Waerden's *Moderne Algebra* (Berlin, 1930).

Although the paper by Steinitz clearly aims at a discussion of the general Galois theory, this part has not been completed by Steinitz. In a second appendix the editors give a short representation of the general theory both for finite and infinite fields. The new edition also contains an index of terms and a picture of Steinitz.

I should like to recommend the book to students of algebra; for teachers of advanced algebra it would make a very suitable nucleus for a short seminar on abstract fields.

Finally, I shall make an observation without great consequence regarding a remark in the introduction, namely, "The appendix and explanations are mainly due to the younger of the two editors." It seems that this statement does not have the decisiveness expected in mathematics, as it cannot be supposed that all readers have made the personal acquaintaince of both editors. I know that both editors are young men, and it seems necessary that the exact age should be stated.

OYSTEIN ORE

NOTES

A detailed announcement has now been issued by the Organizing Committee in charge of the International Congress of Mathematicians to be held in Zurich, Switzerland, next fall (see this Bulletin, vol. 37, p. 661). The outline of the proposed program is as follows:

September 4:	8 P.M., reception.
September 5:	9 A.M., inaugural meeting.
	11–12 A.M., general lecture.
	3–6 P.M., meetings of the sections.
	Evening, concert.
September 6:	9–12 A.M., general lectures.
	Afternoon, excursions.
September 7:	9–12 A.M., general lectures.
(also 9 and 10)	3–6 P.M., meetings of the sections.
September 8:	excursions.
September 11:	4 P.M., official reception.
September 12:	10–12 A.M., final meeting.

Professor W. B. Carver, of Cornell University, will succeed Professor W. H. Bussey as editor of the American Mathematical Monthly, in January, 1932.

The Indian Mathematical Society will celebrate its "silver jubilee" in 1932. A special commemoration number of its Journal will be published, and a jubilee fund is being collected to advance the interests of that Society.

At the meeting of the London Mathematical Society on November 12, 1931, the following officers and new members of the Council were elected: President, Professor A. C. Dixon; Vice-Presidents, Professor S. Chapman, Professor H. Levy, Mr. T. L. Wren; Treasurer, Dr. A. E. Western; Librarian, Professor H. Hilton; Secretaries, Professor G. N. Watson, Mr. F. P. White; new members of the Council, Mr. M. H. A. Newman and Mr. H. T. J. Norton.

The American Institute of Physics, an agency of cooperation in the interest of physics, has recently been established by the American Physical Society, the Optical Society of America, the Acoustical Society of America, and the Society of Rheology. This Institute will perfect details of organization and arrange for cooperative publication of physics journals, and assist the newspapers to disseminate accurate news of important developments and applications of physics. Henry A. Barton has been made the director, and John T. Tate the advisor on publications.

The Henry Burchard Fine Hall of Mathematics of Princeton University was dedicated with appropriate ceremonies on October 22, 1931. On this occasion papers were read by Professors G. D. Birkhoff and G. A. Bliss, both formerly members of the department of mathematics of Princeton.

The astronomy prize of the Accademia dei Lincei was awarded to Professor G. Abetti, of the University of Florence, not to G. Tobetti, as was announced in the September, 1931, number of this Bulletin. The name of the other recipient of this prize (V. Nobile) is given correctly there.

The Royal Society of London has awarded its Sylvester medal to Professor E. T. Whittaker for his original contributions to pure and applied mathematics, its Hughes medal to Professor W. L. Bragg for his pioneer work on the elucidation of crystal structure by X-ray analysis, and its Copley medal to Sir Arthur Shuster for his distinguished researches in optics and terrestrial magnetism.

The Frederick Ives medal of the Optical Society of America has been awarded to Dr. Theodore Lyman, emeritus professor of mathematics and natural philosophy at Harvard.

The John Fritz gold medal, conferred by the four American societies of civil, mining and metallurgical, mechanical, and electrical engineers, has been awarded for 1932 to Professor M. I. Pupin, of Columbia University, for his achievements as "scientist, engineer, author, inventor of the tuning of oscillating circuits and the loading of telephone circuits by inductance coils."

On the occasion of the opening of the new laboratory of physical chemistry at the University of Freiburg im Breisgau, honorary degrees were conferred on Lord Rutherford, Professor Manne Siegbahn of the University of Uppsala, and Professor V. M. Goldschmidt of Göttingen.

On the occasion of the centenary meeting of the British Association for the Advancement of Science, the University of London conferred honorary doctorates on Lord Rutherford and Sir J. J. Thomson.

Dr. W. Friedli has been appointed to a professorship at the University of Berne, as successor to Professor C. Moser, retired.

Dr. Fritz Kirchner, of the University of Munich, has been promoted to an associate professorship of physics.

Dr. Walther Mayer has been promoted to an associate professorship of mathematics at the University of Vienna.

Professor Hans Mohrmann, of the Darmstadt Technical School, has been appointed professor of mathematics at the University of Giessen.

Dr. Trygve Nagel, of the University of Oslo, has been appointed successor to Professor A. Wiman, at the University of Uppsala.

Professor Emile Picard has been appointed to an honorary professorship in the Faculty of Sciences of the University of Paris.

Professor Michel Plancherel has been appointed rector of the Zurich Technical School.

Professor G. H. Hardy, Mr. H. S. M. Coxeter, and Mr. L. C. Young have been elected to fellowships in Trinity College, Cambridge.

Dr. E. L. Ince has been appointed lecturer in mathematics at the University of Edinburgh.

Assistant Professor C. T. Bumer of the Ohio State University, has been appointed assistant professor of mathematics at Kenyon College.

Colonel C. P. Echols, professor of mathematics at the United States Military Academy at West Point, has retired from the Army.

Dr. Irwin Roman, mathematical physicist of the Geophysical Research Corporation, has been appointed assistant professor of mathematics at the Michigan College of Mining and Technology.

Professor Elijah Swift, head of the department of mathematics in the College of Arts and Sciences of the University of Vermont, has been made dean of that College.

Professor Niels Nielsen, of the University of Copenhagen, is dead.

Professor Charlotte A. Scott, formerly professor of mathematics at Bryn Mawr College, died November 8, 1931, at the age of seventy-three. Professor Scott had been a member of the American Mathematical Society since 1891.

Dr. Steven Marshall Hadley, who was for many years dean and professor of mathematics at Penn College, Oskaloosa, Iowa, died at Whittier, California, on November 10, 1931.

Professor G. O. James, of Washington University, St. Louis, died November 24, 1931. Professor James had been a member of the American Mathematical Society since 1906.

Mr. E. A. Kholodovsky, formerly instructor in mathematics at the Technical School of Petrograd, was killed in an automobile accident in New York City, October 10, 1931. Mr. Kholodovsky had been a member of the American Mathematical Society since 1926.

Professor C. L. E. Moore of the Massachusetts Institute of Technology, died on December 5, 1931, at the age of fifty-five. Professor Moore had been a member of the American Mathematical Society since 1903.

Professor G. W. Myers, of the University of Chicago, died November 23, 1931.

Professor C. W. Watts, of Virginia Military Institute, died on July 10, 1931.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

1. Professor W. L. Ayres: *On a certain neighborhood property.*

The paper considers metric spaces in which the frontier of every proper neighborhood contains at least one ordinary point. An open subset D such that the space contains a point not belonging to $\overline{D}$ is a proper neighborhood. After an examination of the properties of such spaces, the results are applied to obtain new characterizations of the arc and of the simple closed curve. (Received November 7, 1931.)

2. Mr. H. P. Thielman: *An integral addition theorem for Bessel functions.*

In this paper a new integral addition theorem for Bessel functions is derived. The results are obtained by means of a Volterra transformation and its inverse. (Received November 17, 1931.)

3. Dr. Gordon Pall: *On the application of a theta formula to representation in binary quadratic forms.*

It is shown that a certain method of Nazimoff's leads to the number of representations in ax^2+by^2, ab odd, only in the cases $a=1$, $b=1$, 3, 5, 7. Details are supplied for the case $a=1$, $b=7$. (Received November 23, 1931.)

4. Professor H. H. Germond: *A modification of Stirling's formula arising from an asymptotic evaluation of the Wallis integral.*

One method of deriving Stirling's approximation formula is to seek a function, y, whose derivative for each positive integer n is equal to the average rate of change of $n!$ in passing from $n-1$ to $n+1$. The asymptotic expression thus obtained is $y=C(n/e)^n(n+1)^{1/2}$. We determine C as follows. The integral $W(n)=\int_0^{\pi/2}\sin^n\theta d\theta$ has the value $(n!\pi/2)2^n(n/2)!^2$, when n is a positive even integer, and the value $(2^n[(n-1)/2]!^2/2)n!$ when n is a positive odd integer. Taking the geometric mean of $W(n)$ and $W(n+1)$ as a first approximation to the value of $W(n+1/2)$, we obtain a general asymptotic expression $W_1(n)=(\pi/(2n+1))^{1/2}$ for $n>0$. Replacing the factorials in the correct expression for W by the corresponding approximate expressions, and equating W to W_1 leads to a determination of C such that $y=(n/e)^n(2\pi(n+1/6))^{1/2}$. The values of y thus computed agree with the values of $n!$ to within one-half of one per

cent, even when $n=1$. By further steps, a simple interpolation formula is derived which expresses $\Gamma(n+\epsilon)$ in terms of $\Gamma(n)$. (Received November 28, 1931.)

5. Professor J. B. Coleman: *Another coefficient of linear correlation between two variables.*

The method of least squares is applied to a single line of best fit to two variables. A coefficient is obtained, invariant with respect to change of units on the axes, as well as with respect to rotation of the axes. The coefficient reduces to a simple function of the Pearson product-moment coefficient, $1-((1-|r|)/(1+|r|))^{1/2}$, in which r is the Pearson coefficient. (Received November 19, 1931.)

6. Mr. C. H. Fischer: *On certain non-normal correlation surfaces.*

The present paper is concerned with the correlation surface $u=F(x, y)$, where the variables x and y are, respectively, the sum of m and of n independent values of a variable t, and have k values of t in common. The variable t is characterized by a rather arbitrary continuous distribution function, $f(t)$. The general equation of the correlation surface is expressed in terms of iterated integrals. The regression of each variable on the other is linear, and the equations of the regression lines involve only a single integral. The coefficient of correlation between x and y, which is found to be independent of the distribution function, $f(t)$, is $k/(mn)^{1/2}$. These results are extended to more than two variables, where each variable, x_i, after the first, consists of the sum of some k_i constituent values of t selected at random from the preceding variable, x_{i-1}, plus the sum of some (n_i-k_i) values of t, each selected at random directly from the parent distribution function $f(t)$. (Received November 23, 1931.)

7. Professor H. L. Rietz: *On the Lexis theory and the analysis of variance.*

In a paper published in 1921 on a generalization of the Lexis theory for the classification of statistical series with reference to their dispersion into Bernoulli, Poisson, and Lexis types, Coolidge based much of his reasoning on the fundamental dispersion theorem of n variates. In setting up criteria for the practical classification of actual statistical series, Coolidge followed the customary procedure of introducing approximations by replacing $(n-1)/n$ by unity. While certain estimates of population variance given by R. A. Fisher seem to have been obtained as inferences based largely on the number of degrees of freedom of the variates rather than by even moderately rigorous formal mathematical proofs, it is first shown in the present paper that the reasoning of Coolidge leads to exactly the same estimates if we avoid the approximation to which reference is made above. The paper next considers the appropriateness of different estimates of variance for particular purposes in the analysis of the inner structure of data classified into rows and columns. (Received November 16, 1931.)

8. Dr. A. T. Craig: *The simultaneous distribution of mean and standard deviation in small samples.*

The simultaneous distribution function of the arithmetic mean $\bar{x}$ and standard deviation s of samples of n items drawn from a normal universe is known, and is of the form $F(\bar{x}, s) = \phi(\bar{x})\psi(s)$. In the present paper, several theorems are established which make possible, in case of small samples, the determination of $F(\bar{x}, s)$ when the parent population, characterized by a continuous frequency function $f(x)$, is of a rather arbitrary type. The correlation surfaces thus obtained possess some interesting properties. (Received November 24, 1931.)

9. Professor H. H. Germond: *A closed expression approximating to the value of the probability integral.*

The solution of certain problems might be aided, could the value of the probability integral, $I = (2/\pi^{1/2})\int_0^s e^{-x^2}dx$, be expressed in closed terms. I^2 may be viewed as the integral of $(4/\pi)e^{-r^2}$ (where $r^2 = x^2 + y^2$) taken over the square bounded by the x and y axes and the lines $x = z$ and $y = z$. Let P^2 be the integral of the same function, but taken over the quadrant of a circle of the same area as the square and with its center at the origin. Then $P = (1 - e^{-4z^2/\pi})^{1/2}$. The values of P are only slightly greater than the corresponding values of I, the maximum difference being less than three-quarters of one per cent, and occurring in the neighborhood of $z = 1.1$. For values of z greater or less than this, the two functions converge. The difference between the two functions may be written in the form $az^3e^{-bz^2} + \cdots$. (Received November 28, 1931.)

10. Dr. A. T. Craig: *On the correlation between certain averages.*

This paper is concerned with the correlation functions of certain averages of a statistical variate x which obeys a given continuous law of probability $f(x)$. The averages considered in the present paper are the arithmetic mean, the median, and the range. The correlation functions of the mean and median and of the mean and range are given for small samples; that of the median and range is given for samples of arbitrary size. (Received November 24, 1931.)

11. Professor E. L. Dodd: *Frequency laws showing stability with reference to the geometric mean and other means.*

A large class of means may be constructed with the aid of a continuous increasing function $y = \xi(x)$ and its inverse $x = \xi^{-1}(y)$ by taking for the mean, X, of n variates $x_1, x_2, \cdots, x_n$, $X = \xi^{-1}[(1/n)\Sigma_1^n \xi(x_i)]$. It is then possible to carry over to these means much that is known regarding stability with respect to the arithmetic mean. If the logarithm of the characteristic function, in the sense of Levy, is $-c|t|^\alpha$, with $0 < \alpha \leqq 2$, and if $\xi(x)$ is an odd function, then from Probability $\{|x_i| < v\} = F[\xi(v), \alpha, c]$, $i = 1, 2, \cdots, n$, we can conclude that Probability $\{|X| < v\} = F[n^{1-1/\alpha}\xi(v), \alpha, c]$. To obtain a geometric mean, G, we take $\xi(x) = \log x$. Then if a is a positive constant, and if Probability $\{a/v < x_i < av\} = F[\log v, \alpha, c]$, $v \geqq 1$, it follows that Probability $\{a/v < G < av\} = F[n^{1-1/\alpha} \log v, \alpha, c]$. (Received November 28, 1931.)

12. Dr. S. S. Wilks (National Research Fellow): *Distributions of certain statistics in matched pairs of samples from a normal bi-variate population.*

The object of this paper is to investigate the distributions of population

parameters estimated by the method of maximum likelihood for a frequently used type of sampling of the following description. A sample of N items is drawn at random from a normal population of two variables x and y, followed by a second sample of N items restricted only in that its distribution of x's shall be identical, item by item, with that of the other. The joint distribution and moments of the estimated variances and covariance were found by means of Romanovsky's integral equation method. Two different estimates can be obtained for the variance of the difference between the two sets of y's; the one from the two sets of y's alone and the other from the estimated variances and covariance of the population of x and y. The latter estimate is found to be distributed with smaller variance than the former. Similar results were found in the distribution of the two estimates of "Student's" ratio. (Received November 20, 1931.)

13. Dr. S. S. Wilks (National Research Fellow): *Determination of characteristic functions of quadratic statistics by determinants.*

It has been shown by Karl Pearson (Royal Society Philosophical Transactions, (A), vol. 200 (1903)) and others that $\int_{-\infty}^{\infty} e^{-\theta} dV = \pi^{n/2}\Delta^{-1/2}$, where θ is a positive definite quadratic form in n variables, Δ the determinant of the symmetric matrix of θ, and dV the product of the n differentials, the integral being taken over the entire n-dimensional space. By definition, the characteristic function of the quadratic form w for a sample of n items from a normal population is proportional to $\int_{-\infty}^{\infty} e^{-\theta+\alpha w} dV$, which, for sufficiently small values of α, can be evaluated from the determinant of the matrix of $(\theta - \alpha w)$. The extension of this method to any finite number of quadratic statistics in any finite number of variates in any finite sample is immediate. It is also applicable to linear forms and combinations of the two types of statistics. The determinants arising in connection with variances and covariances are of a form that proved to be easily evaluated, thus affording a comparatively simple determination of the characteristic functions reached by Romanovsky (Metron, vol. 5 (1925), No. 4, and vol. 7 (1927), No. 3) by direct integration. (Received November 20, 1931.)

14. Mr. Albert Wertheimer: *A generalized error function.*

Given a set of numbers l_i, $i = 1, 2, \cdots, n$, obtained by making n observations under the same conditions on a single quantity, l. We wish to define a certain pair of functions $f(p, l_i)$ and $\phi(p, l_i)$ such that the equation $f(p, l_i) = 0$ defines p as a unique value assigned to the observed quantity l, and $\phi(p, l_i)dl_i$ gives to within infinitesimals of higher order the probability that the observed value will lie in the interval bounded by l_i and $l_i + dl_i$. In this paper, we assume that $\Sigma(\partial/\partial p) \log \phi(p, l_i) = 0$ and $f(p, l_i) \equiv p - F(l_1, l_2, \cdots, l_n)$ where F is continuous and single-valued with continuous derivatives up to the second order. We also assume that F is a symmetric function, $F_{l_i} = F_{l_i}(F, l_i)$, and $F_{l_i} \neq 0$ in the region considered. A general error function is then derived from which the so-called normal error law, the Poincaré function, and many other forms of the error function as well as the Pearson curves are derived as special

cases. We also show that there exists a certain function $g(p, l_i)$ uniquely determined from F, and if the assumption is made that $\phi(p, l_i) \equiv \phi\{g(p, l_i)\}$, we obtain as a special case a generalized normal error function, namely $\phi\{g\} = ce^{-k^2g^2}$. (Received November 17, 1931.)

15. Professor C. G. Latimer: *On the class number of an Abelian field.*

Let F be an algebraic field of odd prime degree e, and a primary divisor of the field defined by a primitive Mth root of unity. If M is a prime, it is known that the class number of F is $h = \pm \delta/R$, where R is the regulator of F and δ is a determinant obtained from R by replacing the fundamental units by a unit η of F and $e-2$ of its conjugates (Fueter, Journal für Mathematik, vol. 147, p. 183). More generally, h may be written in this form whenever M is prime to e. In this paper, we show that if h may be so written, then it is the norm of an ideal in the field defined by a primitive eth root of unity. Eisenstein obtained an equivalent result for the case $e=3$, M a prime (Journal für Mathematik, vol. 29, p. 49). We also show that a necessary and sufficient condition that F contain a unit η such that η and $e-2$ of its conjugates form a fundamental set is that the above mentioned ideal be principal. (Received November 27, 1931.)

16. Professor B. W. Jones: *On the reduction of positive quaternary quadratic forms.* Preliminary communication.

L. Charve (Annales Scientifiques de l'Ecole Normale Supérieure, (2), vol. 11 (1882), pp. 119–134), seeking to define a reduced positive quaternary quadratic form, uses a method similar to that which he used for ternaries. He associates with f the form $\phi = a(x-y)^2+b(x-z)^2+c(x-t)^2+d(x-u)^2+e(y-z)^2+f(y-t)^2+g(y-u)^2+h(z-t)^2+k(z-u)^2+l(t-u)^2$. He calls f reduced when (roughly) not more than two of the coefficients of ϕ are negative. For example, when none are less than zero he says that all ϕ equivalent to a given ϕ are identical except for permutations of *variables*. They are in reality identical except for a permutation of *coefficients* $a, b, \cdots$, which for such a form is quite a different thing. This and more important inaccuracies are corrected in this paper, and progress is made toward defining a unique reduced form f. (Received November 24, 1931.)

17. Dr. L. A. Dye and Professor F. R. Sharpe: *The quartic space involutorial transformations with a double conic.*

A conic C_2, a rational quartic C_4, meeting C_2 in 4 points and a point O, determine a web of quartic surfaces having C_2 as double curve and passing through C_4 and O, so that Cremona transformations of this type exist. They may be regarded as the product of the quadratic inversion $C_2, O \equiv S$, and a cubic transformation T having C_2 and C_4 as a fundamental composite sextic curve of genus 3, so that T can be defined by 3 bilinear equations. In this paper the conditions are found that ST shall be involutorial, and also the actual equations of the transformations when these conditions are satisfied. (Received November 18, 1931.)

18. Dr. M. M. Slotnick: *On fundamental transformations of surfaces.*

By means of the analytic method for the study of the projective differential geometry of conjugate nets discussed in a recent paper by the author (American Journal of Mathematics, vol. 53, No. 1, Jan., 1931) which defines an arbitrary harmonic congruence to the net and its dual, an arbitrary conjugate congruence to the net, the general transformation F can be set up. This method, because of the symmetry of the parameters and other advantages, offers another means of studying the properties of these transformations. (Received November 23, 1931.)

19. Professor L. R. Ford: *Isometric properties of projective transformations.* Preliminary communication.

A projection involving two independent complex variables may be considered as a one-to-one transformation of the points of a four-dimensional real space. The locus of a point in the neighborhood of which volumes are unaltered is a hypercylinder, which will be called the *isometric* hypercylinder. Volumes within this hypercylinder are magnified, those without are diminished. The special connections of this hypercylinder with the geometry of the projection are being studied. For example, the product of the distances of the three fixed points from its axis is equal to the cube of its radius. It is hoped that the isometric hypercylinder will be of use in the study of properly discontinuous groups of projections. (Received November 28, 1931.)

20. Professor Nathan Altshiller-Court: *On the isodynamic points of four spheres.*

The isodynamic points W, W' of four spheres (A), (B), (C), (D) are, by definition, the points common to the six spheres of similitude of the given spheres taken in pairs. The four inverses of $W(W')$ with respect to (A), (B), (C), (D) determine a sphere (R') concentric with the orthogonal sphere (R) of the given spheres, and W is a center of similitude of (R') and the sphere (O) determined by the centers A, B, C, D of the four given spheres. If O, R are the centers, and r, s the radii of the spheres (O), (R), we have $s^2=(1-k)\cdot(r^2-k\cdot OW^2)$, where $k=OR:OW$. These properties are readily applicable to a tetrahedron, for the centroid G of a tetrahedron $ABCD$ is an isodynamic point of the four spheres having their radii proportional to the medians of $ABCD$ and for centers points dividing these medians in the same ratio. As a further application the variable tetrahedron is considered which is inscribed in a fixed sphere and has a fixed centroid. The spheres having for diameters the medians of this tetrahedron are orthogonal to a fixed sphere having for its center the Monge point of the tetrahedron. (Received November 28, 1931.)

21. Professor C. O. Oakley: *Semi-linear equations. Part* II: *Irregular polygons.*

The author continues the study of semi-linear equations and their geometric interpretations. A semi-linear equation is of the form $u_0+m_1|u_1|+\cdots+m_n\cdot|u_n|=0$, where $u_i=a_ix+b_iy+c_i$, $i=0, 1, \cdots, n$; it is said to be of order

$p(\leq n)$ when there are exactly p non-vanishing coefficients among the m's. Problems of the representation of certain broken-line curves, polygons, polygonal areas, etc., by means of semi-linear equations of restricted order are treated. For equations of small order, the inverse problem is also treated. (See abstract 179 in this Bulletin, May, 1931, for Part I: Regular polygons.) (Received November 28, 1931.)

22. Professor H. C. Shaub: *Plane Cremona transformations arising from a generalization of the transformation by reciprocal radii.*

The generalizations treated in this paper arise from an equation of the form $r\,r_1 = R(x, y)$, where R is a rational function and r, r_1 the radii vectores of a point P and its image P_1. The transformations are of the perspective deJonquières type. Certain modifications, giving rise to involutions, are also considered. (Received December 2, 1931.)

23. Professor A. D. Campbell: *Apolarity in the Galois fields of order 2^n.*

In this paper is derived the equation in hyperplane coordinates of the quadratic $\Sigma a_{ij}x_ix_j=0$, where $i, j=1, 2, \cdots, m$, where $j \geqq i$ and $a_{ji}=0$ if $j \neq 0$, also where a_{ij}, x_i, x_j all belong to the Galois fields of order 2^n. This equation in hyperplane coordinates differs according as m is even or odd. The discriminant Δ of $\Sigma a_{ij}x_ix_j=0$ is the determinant with rows $0, a_{12}, \cdots, a_{1m}$; $a_{12}, 0, \cdots, a_{2m}; \cdots; a_{1m}, a_{2m}, \cdots, 0$, if m is even, and one-half the determinant with rows $2a_{11}, a_{12}, \cdots, a_{1m}$; $a_{12}, 2a_{22}, \cdots, a_{2m}; \cdots; a_{1m}, a_{2m}, \cdots, 2a_{mm}$, if m is odd. Applying Δ to the pencil of quadratics $\lambda\Sigma b_{ij}x_ix_j+\mu\Sigma a_{ij}x_ix_j=0$) we get the condition for apolarity, which differs according as m is even or odd. The geometric interpretation of apolarity is different in the Galois fields of order 2^n from what it is in the ordinary real and complex domains or in the Galois fields of order p^n for $p>2$. (Received November 23, 1931.)

24. Professor A. A. Albert: *Normal division algebras of degree 2^e over an algebraic field.*

The author considers normal division algebras A of degree n over an algebraic field $R(x)$ for the case where n is a power of 2. The exponent of A is the least integer r such that A^r (the direct product power) is a total matric algebra. He proves that $r=2^e=n$ for this case. He also shows that the only self-reciprocal normal division algebras over an algebraic field are normal division algebras of order unity or four. As a consequence we have the very important theorem that the multiplication algebra of every pure Riemann matrix of the first kind is a field or a generalized quaternion algebra over a field. (Received November 17, 1931.)

25. Dr. Gordon Pall: *Partitionment of representations in indefinite binary quadratic forms.*

Representations of an integer in an indefinite binary quadratic form may be grouped into *sets* according to whether they are or are not transformable into

one another by integral automorphs of the form. Assuming the least positive solution (t_1, u_1) of $t^2-du^2=4$, we obtain intervals containing one and only one representative of any given set of solutions of $ax^2+bxy+cy^2=n$. For example, in each set of solutions there is one and only one (x, y) satisfying $2|an|-2an \leqq dy^2 \leqq t_1|an|-2an$, $y \geqq 0$, except that there may be two solutions (x', y) and (x, y) if y is at the boundary of this interval. As the simplest application let us note that if $n>0$, the number of integral solutions (x, y) of $x^2-2y^2=n$, $|x| \geqq 2y \geqq 0$, is $g(n)=\Sigma(2|\nu)$, where ν runs over the positive divisors of n, except when n is a square or the double of a square, when this number is $g(n)+1$. (Received November 28, 1931.)

26. Dr. Gordon Pall: *A class of universal functions.*

The function $axy+bx+cy+d$, where a, b, c, d are integers, and $a \neq 0$, represents all integers for integral values of x and y, if and only if $b \equiv \pm 1$ or $c \equiv \pm 1 \pmod{a}$, or $a=6$, $b \equiv \pm 3$, $c \equiv \pm 2 \pmod 6$ or vice versa for b and c. A similar theorem holds for such functions which are positive for positive integers x and y and represent all positive integers for such values x and y. (Received November 28, 1931.)

27. Professor R. G. Archibald: *Properties of highly composite numbers.*

A highly composite number has been defined by S. Ramanujan as a positive integer the number of positive divisors of which exceeds that of every smaller positive integer. He has developed a theory of these numbers and obtained for them some properties analogous to the corresponding properties for primes; for example, if x is any quantity not less than unity, there is at least one highly composite number greater than x and not exceeding $2x$ (an analogue to Bertrand's postulate for primes). The present paper shows, on the other hand, that the analogy with primes may break down. It is proved, for example, that the sum of the reciprocals of all highly composite numbers is finite, and that every arithmetical progression $ax+b$, with a and b relatively prime, can contain only a finite number of highly composite numbers. More generally, it is shown that every arithmetical progression $ax+b$, with $a>b \geqq 1$, contains only a finite number of highly composite numbers. (Received November 28, 1931.)

28. Professor I. M. Sheffer: *A type of conformality in three dimensions.*

The class of conformal point transformations in three dimensions being rather restricted, as compared to the 2-dimensional case, one inquires if one can enlarge the class by demanding less. Thus, instead of asking for what transformations will angles between *all* pairs of curves transform conformally, let us put the following question: when will there exist a family of surfaces such that angles between all pairs of curves lying on any member of this family transform conformally? A transformation fulfilling such a condition is (we say) quasi-conformal; in this paper we examine the conditions for quasi-conformality. (Received November 24, 1931.)

29. Professor J. L. Synge: *The apsides of general dynamical systems.*

Hadamard (Journal de Mathématiques, (5), vol. 3 (1897)) discussed the relations of a dynamical trajectory to any system of surfaces $S=$const. in the manifold of configurations. The method, which is based on the determination of the sign of $d^2S/d\rho^2$, is now more simply presented by the use of tensor calculus, and applied to obtain information concerning the distribution of apsides in the motion of systems of some important general types. The method is based on the use of the characteristic function F of the manifold (half the square of geodesic distance). The apsides considered are the points of contact of the trajectory with the equipotential surfaces and with a family of concentric spheres. (Received November 28, 1931.)

30. Dr. C. H. Dix: *A discontinuous dynamics.*

A theory of discontinuous motion on a lattice is developed in which the equations of motion are difference equations. These equations can be given in either the Lagrangian or the Hamiltonian form. In the Hamiltonian form they suggest a theory of canonical transformations and of Pfaffian difference forms which is developed by the author. (Received November 27, 1931.)

31. Dr. Eberhard Hopf (International Research Fellow): *On the time-average theorems in dynamics.*

J. v. Neumann has quite recently proved an important theorem on time-averages in dynamics using the spectral theory of unitary transformations in Hilbert space. Here a simpler and more elementary proof of v. Neumann's theorem is given. Let P denote points of an analytic manifold Ω, and let $P_t=T_t(P)$, $T_0(P)=P_0$, $T_sT_t=T_{s+t}$ be an analytic one-parameter group of $(1, 1)$ transformations of Ω into itself. A Lebesgue measure is assumed to exist on Ω and be invariant under T_t. Then any quadratically integrable function $f(P)$ possesses a time-average, in the sense of convergence in the mean over Ω. The analyticity restrictions are unessential, and the time-average can be expressed as a space-average. (Received November 30, 1931.)

32. Dr. L. M. Blumenthal: *A complete characterization of proper pseudo d-cyclic sets of points.*

In characterizing the n-dimensional euclidean space R_n among general semi-metric spaces, Karl Menger introduced and characterized pseudo-euclidean sets of points (Mathematische Annalen, vol. 100 (1928), p. 120). These are sets of points such that each $n+2$ of them is congruent with $n+2$ points of the R_n, while the whole set is not congruent with a subset of the R_n. Since the R_n has the quasi-congruence order $n+2$, these sets consist of exactly $n+3$ points. In this paper we consider a d-circular space; that is, a space congruent with the convex circle of metrical diameter d. Such a space is *not euclidean*. A pseudo d-cyclic set of points is a set such that each three of the points is congruent to three points of the d-circular space, while the whole set is not congruent with a subset of this space. The set is proper if it does not contain a convex tripod. We futher assume that in case four points of a set are pseudo-

linear, two of these points are diametral. The theorems contained in this paper completely characterize such sets. The circular space considered has *not* the quasi-congruence order three; and hence these sets do not consist of exactly four points. Pseudo d-cyclic quadruples are characterized, and it is then proved that *a proper pseudo d-cyclic set containing more than four points is equilateral.* (Received November 16, 1931.)

33. Dr. Rothwell Stephens: *Continuous transformations of abstract spaces.*

This paper is a study of continuous transformations on a general range, the topological space. In this space a continuous transformation is difficult to define, since all our intuitive notions cannot be satisfied. The Sierpinski definition seems the most satisfactory. Invariant properties for continuous, bicontinuous, and biunivocal transformations are determined. Finally the following problem is considered: Characterize the most general space such that there exists a non-constant continuous transformation to a given type of space. Necessary and sufficient conditions are found for the existence of continuous transformations of a space to neighborhood, accessible, and L spaces. (Received November 25, 1931.)

34. Mr. Alfred Korzybski: *A non-aristotelian system and its necessity for rigor in mathematics and physics.*

This paper gives an anthropological approach to the subject. Identity, a fundamental, false to fact, postulate of the aristotelian trilogy (the aristotelian, euclidean, newtonian systems), establishes elementalism; a non-aristotelian system rejects "identity," becomes non-elementalistic. The inter-relation between the non-aristotelian, non-euclidean, and non-newtonian systems (called non-systems) is given, also the essential differentiation between the un-speakable objective levels (ordinary objects, processes, action, functionings, immediate feelings, etc.) and verbal levels, establishing *structure*, defined in terms of relations or multi-dimensional order, as the only link between the two worlds. Structure becomes the only content of "knowledge." Semantic definition of number and mathematics in terms of relations explains why mathematics gives structure-knowledge. There is *one name* for happenings on the un-speakable levels and "mental pictures," a partial mechanism of identification. The two-valued aristotelian logic is a limiting case of a general many-valued logic of probability. Two-valued logic does not apply to processes, the objective world, the foundation of mathematics, the theory of infinity, etc. The many-valued logic covers all 1931 human needs, mathematics and mathematical physics included. (Received November 27, 1931.)

35. Professor J. L. Synge: *Thermostatic control.*

This paper studies the oscillations of the temperature of a room, the source of heat being automatically cut off when the temperature rises above an assigned level. The temperature of the room satisfies a linear differential equation of the second order with constant coefficients and with a discontinuous term. The problem has an exact analogue in the motion of a damped harmonic oscillator of the dead-beat type, subject to a constant disturbing force for a certain

range of positions of the particle. It is found that the amplitude and the periodic time of the oscillations both tend to zero. (Received November 28, 1931.)

36. Dr. C. H. Dix: *Generalizations of time in dynamics.*

Possible generalizations of "time in dynamics" are considered. Instead of the Hamiltonian action integral we have a new integral in which the time is replaced by a monotone function $\tau(t)$ of a continuous parameter, t. The velocity is then the Stieltjes derivative of the coordinate $q(t)$ with respect to the $\tau(t)$. In the case of no force, the velocity is constant except for a set, T, of the t's for which $\int_T d\tau(t)=0$. The case where the potential is not zero has also been considered. (Received November 27, 1931.)

37. Dr. H. B. Curry (National Research Fellow): *Some additions to the theory of combinators.*

The present paper contains two amplifications of the author's dissertation *Grundlagen der kombinatorischen Logik* (American Journal of Mathematics, vol. 52 (1930), pp. 509–536, 789–834). These are (1) a revision of the definition of normal form so that to every normal sequence there will correspond a unique normal combinator, (2) a generalization of the principal theorems on equality of combinators, so that the restriction that the combinators be "eigentlich" is removed. (Received November 25, 1931.)

38. Professor R. G. Putnam: *On sets of constant dimension.*

In this paper several theorems concerning constant dimensional sets (Menger, *Dimensionstheorie*, pp. 308–309) of separable spaces are proved. In particular, it is shown, for certain subsets of a compact separable space, that a necessary and sufficient condition for constancy of dimension is that each set be contained in a subset of the same dimension, which is both an F_σ and a G_δ. (Received November 23, 1931.)

39. Dr. Hassler Whitney (National Research Fellow): *Conditions that a graph have a dual.*

It is shown by purely combinatorial means that a graph has a dual if and only if it contains neither of two certain graphs as subgraphs. This, with a theorem of the author (Proceedings of the National Academy, vol. 17 (1931), p. 127, Theorem 15), gives a proof involving little of a point-set nature, of the case for finite graphs of a theorem of Kuratowski (Fundamenta Mathematicae, vol. 15 (1930), pp. 271–283). (Received November 28, 1931.)

40. Dr. Hassler Whitney (National Research Fellow): *Basic graphs.*

R. M. Foster has classified graphs of nullity $\leqq 6$, using the concept of basic graphs and a type of equivalence we shall call semi-homeomorphism. G is semi-homeomorphic with G' if it can be carried into G' by operations of subdividing and recombining arcs, altering the arrangement of the components of the graph, and breaking part of the graph away at two vertices, turning it around, and replacing it. A basic graph is a non-separable cubic graph not semi-homeomorphic with any graph with fewer arcs. All non-separable graphs can be obtained

from basic graphs by dropping out arcs, at the same time letting their end vertices coalesce, and employing the above operations. A basic graph can be obtained from a basic graph of nullity one less by joining the mid points of two arcs by an arc. These theorems and others are proved by the author. (Received November 28, 1931.)

41. Dr. Hassler Whitney (National Research Fellow): *A set of topological invariants for graphs.*

The numbers in question are $p_i=\sum_j(-1)^{i+j}m_{R-j,N-i}$ (see a note by the author in the Proceedings of the National Academy, vol. 17 (1931), pp. 122–125; Theorem 8 should state $m_{ij}'=m_{R-i,N-i}$). On account of the equation just noted, we have, if G and G' are duals, $p_i=m_i'$, $m_i=p_i'$. An interpretation of the p_i is found analogous to that of the m_i in terms of broken circuits. (Received November 28, 1931.)

42. Dr. Hassler Whitney (National Research Fellow): *Characteristic functions and the algebra of logic.*

If A' is any subset of a set R', the characteristic function A of A' is defined to be 1 in A', and 0 in $R'-A'$ (de la Vallée Poussin, *Intégrales de Lebesgue*, p. 7). The characteristic function of $R'-A'$ is $1-A$, that of the common part of A' and B' is AB, and that of the sum of A' and B' is $A\dot{+}B=A+B-AB$. By means of these formulas, the ordinary relations in the logic of classes are easily worked out. Three normal forms for characteristic functions are studied. The notation and normal forms are important in other work of the author. (Received November 28, 1931.)

43. Professor F. R. Bamforth: *Divergent series and singular points of ordinary differential equations.*

In 1929 Professor Birkhoff and the author published under the above title a paper in which the setting of the problem in question was given and the statement was made that difficulties seemed to arise when any attempt was made to apply their method of solution to systems of differential equations of order greater than the third. In the present paper another method of solving this problem is given, such that it can readily be applied to systems of differential equations of any finite order. The problem for the third order is, by means of a transformation of class C^∞ on the dependent variables only, to reduce to the system of differential equations $dz_i/dt=m_iz_i$, $i=1, 2, 3$, every system of differential equations of the type $dx_i/dt=m_ix_i+P_i(x_1, x_2, x_3)$, $i=1, 2, 3$, in which (i) the P_i are power series in their arguments which have no terms of degree less than two and converge for $|x_1|, |x_2|, |x_3|\leqq r$, $r>0$, (ii) the m_i are real constants, $m_1>0$, $m_2>0$, $m_3<0$, which satisfy no equation of the form $m_i=p_1m_1+p_2m_2+p_3m_3$ with any set whatsoever of positive integers p_1, p_2, p_3. (Received November 25, 1931.)

44. Professor W. H. Wilson: *A functional equation involving well known special cases.*

The addition theorems of sine and cosine, and the first two Cauchy equa-

tions are special cases of (1) $f(x+y)=af(x)f(y)+bf(x)g(y)+cf(y)g(q)+dg(x)g(y)$, where a, b, c, d are constants, and x and y are independent variables. This paper is designed to show how the functions $f(x)$ and $g(x)$ that satisfy equation (1) are related to functions that satisfy the four special cases named above. It is assumed that $f(x)\neq 0$, but no such restriction as boundedness, continuity, differentiability, or integrability is assumed. In certain cases $g(x)$ is a constant multiple of $f(x)$ and in these cases the functions $f(x)$ and $g(x)$ are constant multiples of the function that satisfies the exponential equation. In the remaining cases the functions $f(x)$ and $g(x)$ lead to simultaneous equations from which we find that $f(x)$ and $g(x)$ are expressible in terms of functions satisfying the four equations named at the beginning. (Received November 23, 1931.)

45. Professor H. T. Davis: *A singular case of the Laplace differential equation of infinite order.*

In this paper the author considers the inversion of the Laplace differential equation of infinite order, $\sum_{n=0}^{p} A_n(d/dx)\rightarrow u(x)=f(x)$, $p>0$, for the singular case where $A_p(0)=0$. It is shown that the difficulties of inversion, which have been formally removed by a transformation due to O. Perron (Mathematische Annalen, vol. 84 (1921), p. 41), reside essentially in the nature of the operators $z^{-\nu}$ and $z^{-\nu}\log^m z$, $z=d/dx$. Adjoin these branch-point operators to the operators of ordinary integration and differentiation, and it is possible to give a complete account of the inversion of the equation. Examples are cited from the theory of integral and difference equations to illustrate the application of these operators. (Received November 27, 1931.)

46. Professor J. M. Thomas: *Pfaffian systems of species one.*

If the minimum number of differentials in terms of which a Pfaffian system of r equations can be expressed is $r+s$, the integer s is the *species* of the system. Completely integrable systems are of species zero. The next systems in order of simplicity are those of species one. It is shown how to recognize such a system and how to reduce it to a form involving the minimum number of differentials. The latter operation only requires the solution of ordinary differential equations. When the class of the system plus the number of equations in the maximum system of species zero contained by it is $2r+1$, the system admits a canonical form. (Received November 27, 1931.)

47. Professor Morris Marden: *A generalization of Weierstrass's and Fekete's mean-value theorems.*

The two principal results of the present paper may be stated as follows. (1) If on a curve $C: z=k(t)$, $a\leqq t\leqq b$, $f(z)$ and $g(z)$ assume any values within a given convex region F and an angular domain G (with vertex at $w=0$ and magnitude $\gamma<\pi$) respectively, the point σ as defined by the equation $\int_a^b f(z)\cdot g(z)dt=\sigma\int_a^b g(z)dt$ lies in the star-shaped region consisting of all points at which F subtends an angle $\phi\geqq\pi-\gamma$. Weierstrass's theorem is the case $\gamma=0$. (2) If $P(z)$ is a polynomial of degree n, and k_j and α_j are points of a convex region K and an angular domain A (with vertex at $z=0$ and magnitude $\gamma<\pi$) respectively, $P(z)$ assumes the value σ, as defined by the equation $\sum_1^m \alpha_j P(k_j)=\sigma\sum_1^m \alpha_j$, at least once in the star-shaped region consisting of all points at

which K subtends an angle of $\phi \geqq (\pi-\gamma)/n$. When $\gamma=0$ and $m=2$, the theorem coincides with Fekete's, and when $\gamma=0$ (no restriction on m) it gives a somewhat better approximation to the position of σ than does Nagy's extension of Fekete's theorem. (Received November 27, 1931.)

48. Dr. I. J. Schoenberg: *Concerning a certain class of linear inequalities connected with the theory of completely monotonic functions.*

It is well known that the most general solution of a system of linear homogeneous inequalities may be represented as a linear homogeneous combination with non-negative coefficients of a certain set of so-called *fundamental solutions*. This fact suggests that some infinite set of linear inequalities in infinitely many variables may have its general solution representable as Stieltjes integrals of the type $x_n=\int_a^b \phi_n(t)d\chi(t)$, $n=0, 1, 2, \cdots$, with $\phi_n(t)$ continuous and $\chi(t)$ non-decreasing. This is indeed the case for completely monotonic sequences according to a theorem of Hausdorff. In a previous note *On finite and infinite completely monotonic sequences* (to appear in this Bulletin) the author has derived from this point of view Hausdorff's theorem mentioned above. In the present paper the same point of view is applied to obtain Hausdorff's results (Mathematische Zeitschrift, vol. 9, pp. 280–299) concerning his more general *sequences of moments* (Momentfolgen) and his *functions of moments* (Momentfunktionen) which include the so called Bernstein-Widder theorem on completely monotonic functions. These results are obtained by the use of a particular case of a method which may have further applications. (Received November 30, 1931.)

49. Professor L. W. Cohen: *A type of non-absolutely convergent integral.*

Let $f(x)$ be measurable on M, and $f^+(x)$, $f^-(x)$ be the positive and negative parts of $f(x)$. Consider $f_y^+(x)=\min[f^+(x), y]$, $f_y^-(x)=\min[f^-(x), y]$, $y \geqq 0$. Then $\int_M f^+(x)dx=\lim_y \int_M f_y^+(x)dx \leqq +\infty$, $\int_M f^-(x)dx=\lim_y \int_M f_y^-(x)dx \leqq +\infty$. The Lebesgue theory is concerned with the case where the inequalities hold in both cases. It is analogous to the theory of unconditionally convergent series. If the equalities hold in both cases, we have a conditionally convergent integral in the sense that, for any number A, there are sequences y_n', y_n'' such that $\int_{M(y_n'<y_n'')} f(x)dx=\lim_n [\int_M f_{y_n'}^+(x)dx-\int_M f_{y_n''}^-(x)dx]=A$. As a normal form of this definition we put $\int_M f(x)dx=\lim_y \int_M f_y(x)dx$, provided the limit exists, where $f_y(x)=f_y^+(x)-f_y^-(x)$. It is easily shown that this definition includes that of Lebesgue, that $\int_{-1}^{1}(1/x)dx=0$, that $g(y)=\int_M f_y(x)dx$ is uniformly continuous on $0 \leqq y < +\infty$, and that the "Hölder means" $\lim_{y_n}(1/y_n)\int_{y_n} dy_{n-1}\cdots(1/y_1)\cdot\int_0^{y_1}dy\cdot\int_M f_y(x)dx$ are regular summation processes in a sense analogous to that for non-absolutely divergent series. The paper also includes theorems giving necessary and sufficient conditions for the interchange of limit and integral under the above definition. (Received November 28, 1931.)

50. Dr. Nat Edmonson: *Note on the stereographic definition of plurisegments on the hypersphere.*

In a previous paper (Annals of Mathematics, (2), vol. 31 (1930), p. 13) the author of this note generalized a paper by Professors Evans and Bray (American Journal of Mathematics, vol. 49, p. 153) from three to n dimensions. In this generalization use was made of configurations on the hypersphere which were known as polar caps. The number of such polar caps for a given hypersphere depended on the dimension number of the hypersphere. The purpose of this note is to give an orthogonal parametric representation of the hypersphere such that the number of polar caps per hypersphere is reduced to one for all dimensions. This is accomplished by extending ordinary stereographic projection to n dimensions. All proofs in this new system are essentially the same as in the previous system of representing the hypersphere. (Received November 23, 1931.)

51. Dr. R. P. Agnew (National Research Fellow): *On Riesz and Cesàro methods of summability.*

Let (1) $\alpha_n = n^{-r}\sum_{k=0}^{n-1}(n-k)^r u_k$, (2) $\beta(t) = t^{-r}\sum_{k\leq t-1}(t-k)^r u_k$, (3) $\gamma(t) = t^{-r}\cdot\sum_{k<t}(t-k)^r u_k$, and let σ_n denote the nth element of the Cesàro transform of order r. Then $\sum u_n$ is summable (1), (2), (3), or C_r to L if $\lim_{n\to\infty}\alpha_n = L$, $\lim_{t\to\infty}\beta(t) = L$, $\lim_{t\to\infty}\gamma(t) = L$, or $\lim_{n\to\infty}\sigma_n = L$. In each case r may be any complex constant except that when C_r is involved it may not be a negative integer. We show that, when $\mathcal{R}(r) < 0$, the familiar Riesz definition (3) does not constitute a useful method of summability, and that (2) is equivalent to C_r when $-1 < \mathcal{R}(r) < 0$. The following result, which is of interest in itself, is used in our proof: If $\mathcal{R}(r) < 0$ and $\lim_{n\to\infty} n^{-r}u_n = 0$, then $\lim_{n\to\infty}(\alpha_n - \sigma_n) = 0$. Further relations between (1), (2), (3), C_r, and closely related methods of summability, are obtained. (Received November 28, 1931.)

52. Dr. Hassler Whitney (National Research Fellow): *Note on Perron's solution of the Dirichlet problem.*

A simple proof of the theorem that the lower bound of all upper functions is harmonic is given (see Remak, Journal für Mathematik, vol. 156 (1926), p. 227). (Received November 28, 1931.)

53. Dr. D. G. Bourgin: *The problem of the sheet.*

The term "sheet" characterizes the limiting case of the thin plate under displacements large compared with the thickness when the bending stresses may be neglected. It is first shown that Hencky's (Zeitschrift für Angewandte Mathematik, vol. 1, pp. 81, 423) numerical solution by means of difference equations is invalid because of incorrect boundary conditions. An approximation method is developed based on expressing the non-linear equations as non-homogeneous linear equations. A direct treatment is made possible by the use of the stresses as the primary dependent variables rather than displacements which are usually used in elasticity problems. The solution involves the introduction of a harmonic function determined by the value of a sufficiently well behaved second derivative on the boundary (except for terms in xy and linear terms). The key boundary condition introduced for the clamped rectangular sheet is $\partial T_2/\partial y = \partial S_1/\partial y$ for the boundary $x=0$, $x=a$, and a similar

condition for $y=0$, $y=b$ (Love's notation). The analysis is in terms of the Green's function and the associated orthogonal functions. (Received December 4, 1931.)

54. Professor E. V. Huntington: *A new set of independent postulates for the algebra of logic.*

An abstract of this paper will appear in the February issue of the Proceedings of the National Academy of Sciences.

55. Professor H. A. Simmons: *Generalizations of the author's earlier results relative to the Diophantine equation* $\sum(1/(x_1x_2, \cdots, x_r))=b/a$.

Using notation of our earlier paper (Abstract 37–5–190), we first consider the equation (1) $\Sigma_{n,r}(1/x)+\lambda_{r+1}\Sigma_{n,r+1}(1/x)+\ , \cdots , +\lambda_s\Sigma_{n,s}(1/x)=b/a$, $a\equiv(c+1)b-1$, where r, s, n are positive integers with $r<s\leqq n$; b, c are positive integers; and the $\lambda_t(t=r+1, \cdots, s)$ are integers $\geqq 0$. Application of Kellogg's process (American Mathematical Monthly, Aug.–Sept., 1921, p. 300) to (1) leads to a solution in positive integers of (1), namely $x=w$, where (2) $w_t=1$ $(t=1, \cdots, r-1)$, $w_r=c+1$, $w_{t+1}=a[\Sigma_{t,t-r+1}(w)+\lambda_{r+1}\Sigma_{t,t-r}(w)+, \cdots, +\lambda_s\Sigma_{t,t-s+1}(w)]+1(t=r, \cdots, n-2)$, $w_n=a[\Sigma_{n-1,n-r}(w)+\lambda_{r+1}\ \Sigma_{n-1,n-r-1}(w)+, \cdots, +\lambda_s\Sigma_{n-1,n-s}(w)]$. In the cases where (i) $\lambda_t=1(t=r+1, \cdots, s)$, (ii) λ_{r+1} is an arbitrary positive integer and $\lambda_t=0$ for $(r+1)<t\leqq s$, we obtain for the w of (2) two theorems on maximum numbers which are analogous to results that were stated in the abstract of the previous paper. We also obtain the Kellogg solution of the equation $\Sigma_{t=r}^{s}[\lambda_t\Sigma_{n,t}(1/x)]=1$, where λ_r is a positive integer and $\lambda_t(r<t\leqq s)$ is an integer $\geqq 0$; and for this equation we obtain results analogous to those described for (1). (Received December 28, 1931.)

56. Mr. E. R. Lorch: *Elementary transformations.*

An elementary transformation on two variables of order one is a transformation defined by two elementary functions each of order one at most. Necessary and sufficient conditions are given in this paper that the inverse of an elementary transformation on two variables of order one be elementary and of order one. (Received December 9, 1931.)

57. Professor R. L. Jeffery: *k-dimensional sets in n-dimensional space.*

A paper by A. S. Besicovitch (Mathematische Annalen, vol. 98 (1927), p. 422) deals with the geometrical properties of linearly measurable plane sets of points. These sets are measurable in the sense of Carathéodory, and this fact plays a fundamental rôle in the various discussions. The present paper shows that the results of Besicovitch hold for sets which are not necessarily measurable. It is also shown that k-dimensional sets in n-dimensional space have properties analogous to those of one-dimensional plane sets: e.g., the upper density is not greater than unity, while the lower bound of the upper density is different from zero, and is a function of k. (Received December 3, 1931.)

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

The usual February Meeting in New York City will be omitted.

NEW YORK CITY, March 25-26, 1932.

Abstracts must be in the hands of Associate Secretary Tomlinson Fort, 501 West 116th St., New York City, not later than March 5. All such abstracts will appear in the March issue of the Bulletin. By invitation of the program committee, there will be a symposium on *Summability of series*, at which Professor Einar Hille will speak on *Summation of Fourier series*, Professor W. A. Hurwitz will speak on *General theorems on linear transformation of sequences*, and others will participate.

CHICAGO, April 8-9, 1932.

Abstracts must be in the hands of Associate Secretary Mark H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than March 18. All those received on or before March 6 will appear in the March issue of the Bulletin. By invitation of the program committee, Professor R. L. Wilder will deliver an address entitled *Point sets in three and higher dimensions and their investigation by means of unified analysis situs.*

On account of the fact that the Summer Meeting of the Society in 1932 will be held on the Pacific Coast, the meetings that were to have been held at Stanford University (April, 1932), at Seattle (June, 1932), and at Los Angeles (November, 1932) will be omitted.

LOS ANGELES, CALIFORNIA, SUMMER MEETING AND COLLOQUIUM, August 30-September 2, 1932.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than July 26. Abstracts received by July 7 will appear in the July issue, or an earlier issue, of this Bulletin. Professor J. F. Ritt will deliver a series of colloquium lectures on *Systems of algebraic differential equations.*

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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